Critical phenomena in inhomogeneous excitable media; modeling on a "TV-ANALOG"

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(Submitted 5 June 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **42**, No. 3, 84–87 (10 August 1985)

The creation and shaping of nonlinear structures in 2D nonequilibrium media with inhomogeneities of various types are studied through an analog modeling with a "(television receiver)-camera-feedback" apparatus. It is shown, in particular, that the propagation of excitation fronts in media with homogeneous — on the average — distributed defects is a critical phenomenon in the parameter ρ, the relative density of defects. Specifically, at $\rho > \rho_{cr}$ any spatially bounded excitation will collapse, while at $\rho < \rho_{cr}$ it will expand. The velocity (v) of the excitation front increases with distance from the critical point in a power-law fashion: $v \sim (\rho_{cr} - \rho)^{\gamma}$. The critical index $\gamma \in (0,1)$ is determined by the extent to which the medium deviates from equilibrium. Another effect found is the detachment of excitation "droplets" in media with a smooth inhomogeneity.

1. Reaching an understanding of the role played by inhomogeneities in the formation of steady-state and time-varying structures in nonequilibrium medium and of the effect of inhomogeneities on the propagation, excitation, and interaction of these structures is currently one of the most interesting and most difficult problems of nonlinear physics. Problems of this sort are important in analyzing the motion of domain walls in ferrite films,[1] of oxidation fronts at the surfaces of activators,[2] of flames,[3] of the structures which are produced at laser targets[4] and in nonuniform hydrodynamic flows[5] and of many other topics in other fields of physics and beyond physics.

Phenomena of this type are known to be describable by nonlinear partial differential equations with coordinate-dependent parameters. Solving equations of this sort in the 2D case, even if the geometry of the inhomogeneities is comparatively simple, taxes the capabilities of the computers currently available, forcing a search for new methods for studying these phenomena. In the present study we use an analog modeling device to analyze the nonlinear processes that occur in inhomogeneous excitable media. The apparatus consists of a television receiver which is fed a signal from a television camera, which is in turn pointed at the screen of the receiver. This apparatus, which we call a "TV-ANALOG," was proposed by Abraham[6] for modeling nonlinear media and has been discussed in detail.[7–9] So far, however, no results pertinent to the modeling of processes in specific nonequilibrium media have been obtained.

2. The initial equations for the description of the TV-ANALOG are difference equations averaged over the response time τ_n of the target of the camera. These equations relate the potential u_n at the surface of the target of the camera, at the point (x,y), to the image brightness I_n on the screen of the receiver at the point corresponding to (x,y) at the times $t_n = t_0 + n\tau_k$ (τ_k is the frame formation time; $\tau_m \sim 10\tau_k$):

$$u_{n+1}(\mathbf{x}) = \beta \hat{R}_t(u_n(\mathbf{x})) + f_1[\hat{R}_0(I_n(\mathbf{x}))],$$

$$I_{n+1}(\mathbf{x}) = f_2[u_{n+1}(\mathbf{x})] \quad (\beta = 1 - \tau_k/\tau_m). \tag{1}$$

Here $\hat{R}_0(I_n(\mathbf{x})) = \int_{S_s} R_0(\mathbf{x} + \vec{\zeta} - \vec{\xi})I_n(\vec{\xi})d\vec{\xi}, \hat{R}_t(u_n(\mathbf{x})) = \int_{S_t} R_t(\mathbf{x} - \vec{\xi})u_n(\vec{\xi})d\vec{\xi}$; and S_s and S_t are the surface areas of the television screen and of the camera target, respectively. The function $f_1[...]$ describes the transformation of the optical image into an electronic image, while $f_2[...]$ describes the inverse transformation. The function R_0 determines the blurring of the image in the optical part of the apparatus and is approximately Gaussian, like the function R_t, which characterizes the charge spreading on the target. The parameter $\vec{\zeta}$ is related to the relative displacement of the optical axes of the camera and the receiver. Looking ahead to the modeling of processes with scale times $t \sim l\tau_k$ ($l \gg 10$), and taking an average over the interval, we find the continuous time

$$\tau_k \frac{\partial u}{\partial t} = -(1-\beta)\int_{S_t} u(\vec{\xi})R_t(\mathbf{x} - \vec{\xi})d\vec{\xi}$$
$$+ f_1[\alpha(\mathbf{x})\int_{S_s} f_2(u(\vec{\xi}))R_0(\mathbf{x} + \vec{\zeta} - \vec{\xi})d\vec{\xi}]. \tag{2}$$

The given inhomogeneity $\alpha(\mathbf{x})$ of the parameters of the medium being modeled is produced by means of masks in the plane which is conjugate to the plane of the electronic image. Expanding the operators, and taking into account the linearity of the working region of $f_1(u)[f_1(u) = Ku]$, we find a nonlinear equation with diffusion, which is ordinarily used to describe the propagation and interaction of excitations in nonequilibrium media[1-4] ($\delta^2 = \int_{S_s} (1/2)\vec{\xi}^2 R_0(\vec{\xi})d\vec{\xi}$):

$$\frac{\partial u}{\partial t} = F(u,\mathbf{x}) + j(u,\mathbf{x})\nabla u + \alpha(\mathbf{x})[\delta^2\nabla(D(u)\nabla u) + \vec{\zeta}\nabla(D(u)\vec{\zeta}\nabla u)]. \tag{3}$$

Here $F(u,\mathbf{x}) = -(1/\tau_k - 1/\tau_m)u + (K\alpha(\mathbf{x})/\tau_k)f_2(u)$;

$J(u,\mathbf{x}) = \vec{\zeta}(K\alpha(\mathbf{x})/\tau_k)f_2'(u)$; $D(u) = (K/\tau_k)f_2'(u)$.

The function $F(u,\mathbf{x})$ constructed on the basis of direct measurements is described by an S-shaped curve, with a steepness and mean value determined by the "contrast" and "brightness" adjustments; $f_2'(u)$ is a trapezoidal function.

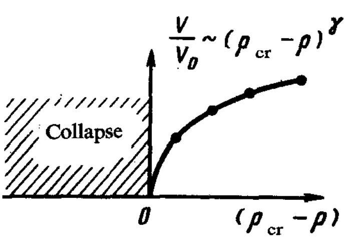

FIG. 1. Velocity of the excitation front versus the relative defect density ρ($\gamma = 0.6$ at $V_0 = 20$ cm/s).

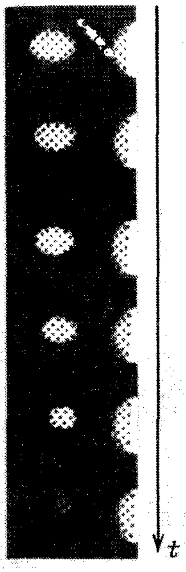

FIG. 2. Collapse (in time) of a spatially bounded excitation in a medium with $\rho > \rho_{cr}$. At the right in this figure we see that the excitation front comes to a halt at the interface between a homogeneous-phase medium with a region containing defects with $\rho > \rho_{cr}$.

For nonequilibrium media, the most typical inhomogeneities are of two types: smooth (determined by, for example, an inhomogeneity of an external field controlling processes in the medium) and random, associated with defects (regions which are not excitable or which have been damaged) and distributed, on the average, homogeneously. Let us examine some particular cases in which these inhomogeneities act independently.

3. In an excitable medium with homogeneously arranged defects one may observe a slowing of the propagation velocity of excitation fronts with increasing defect density. It turns out that this slowing actually occurs, but the phenomenon is critical in nature, similar to percolation phenomena. The results of the experiment are shown in Fig. 1. It was found that there exists a critical defect density ρ_{cr} below which there is a steady-state propagation of excitation fronts (or switching fronts) in an excitable medium with defects, as in a homogeneous medium.[3] If, on the other hand, the density of defects exceeds the critical level, the excitation collapses instead of propagating. Near the critical point, the front velocity varies in a power-law fashion $(\rho_{cr} - \rho)^{\gamma}$, where the critical index $\gamma \in (0,1)$ is determined by the extent to which the medium deviates from

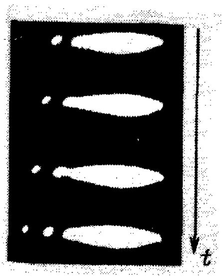

FIG. 3. Periodic detachment of droplets from a localized perturbation in a smoothly inhomogeneous medium with an inhomogeneity gradient above the critical value.

equilibrium (experimentally, by the contrast and the brightness). At $\rho = \rho_{cr}$ we observe steady-state immobile structures. Near the critical point, we observe a similarity: An increase in the "contrast" is equivalent to a decrease in the defect density. At $\rho > \rho_{cr}$, a bounded excitation collapses (Fig. 2).

4. One of the most interesting effects observed in a smoothly imhomogeneous, excitable medium is a periodic detachment of "droplets" from an initially stable excitation region. At a certain critical gradient of the homogeneity, the front of the region of steady-state excitation becomes unstable — it becomes choppy. Droplets form in the nonlinear stage and subsequently break off (Fig. 3). The frequecy at which these droplets break off increases with increasing inhomogeneity gradient.

We wish to emphasize that the TV-ANALOG presents some remarkable opportunities for modeling not only excitable media with diffusion but also a wide variety of nonlinear fields, including problems of 4D field theory.

We wish to thank A. V. Gaponov-Grekhov for useful discussions.

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Translated by Dave Parsons