Boundary potential distribution and quantization of the resistance of 2D structures in the Hall regime

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(Submitted 2 July 1985)

Pis'ma Zh. Eksp. Teor. Fiz. **42**, No. 3, 99–100 (10 August 1985)

Analysis of the boundary conditions for an ideal Hall conductor shows that its resistance is equal to the Hall resistance, regardless of its shape. Corrections for a deviation from the ideal are derived.

It was found in Refs. 1 and 2 that the resistance of 2D structures, R_{SD}, is highly accurately equal to the quantized Hall resistance $R_H = h/ne^2$ in the Hall regime (with a longitudinal conductivity $\sigma_{\parallel} = 0$), where h is Planck's constant, e is the electron charge, and $n = 1,2,3, \ldots$. Fang[3] has offered an explanation of this phenomenon for the case of point conducting contacts (in the absence of short-circuiting effects).

It was shown in Ref. 4 through a direct calculation of the fields that the energy dissipated at each of the electrodes is $W = I^2R/2$, and it was concluded on this basis

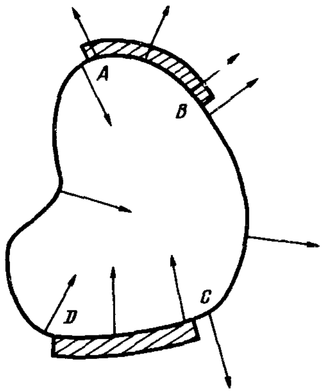

FIG. 1. Sample of arbitrary shape with contacts *AB* and *CD*. The arrows show the direction of the electric field at the boundary of the sample.

that there is a quantization of the resistance of the conducting contacts. In an ideal Hall conductor ($\sigma_{\|} = 0$), however, there is a singularity in the current density at one end of an electrode (and it was there that the energy dissipation was calculated incorrectly), which will generally disrupt the Hall regime (see Ref. 5, for example) and thereby change the field configuration. Furthermore, the methods of Refs. 3 and 4 cannot tell us the difference $R_{SD} - R_H$ which arises because of a deviation ($\sigma_{\|} \neq 0$) from an ideal Hall conductor.

In the present letter we analyze the boundary conditions for an ideal Hall conductor and we show that its resistance is equal to the Hall resistance, regardless of the dimensions and shape of the conductor. We find the difference between R_{SD} and the Hall resistance (measured between Hall electrodes) for a rectangular sample in the nonideal regime.

Let us examine an ideal Hall conductor; see Fig. 1, where *AD* and *BC* are free boundaries, and *AB* and *CD* are electrodes. At the free boundaries the normal component of the current density vanishes, so that we have $\sigma_{\|} E_n + \sigma_H E_t = 0$. Since $\sigma_{\|} = 0$, we have $E_t = 0$ (E_n and E_t are the normal and tangential components of the electric field, and $\sigma_H = R_H^{-1}$ is the Hall conductivity). In the ideal regime, the segments *AD* and *BC* are thus equipotentials.

Electrodes *AB* and *CD* are, obviously, also equipotentials. Regions *AC* and *CA* are therefore equipotentials (the distribution of electric fields at the boundary is shown in Fig. 1). At points *A* and *C* there is a singularity in the electric field, which corresponds to the potential difference across the electrodes (in reality, the high current density disrupts the Hall regime near these points, but this circumstance has no bearing on the discussion below). The potential difference across the electrodes, V, is thus equal to the potential difference across the free boundaries, V_H, so that we have

$$R_{SD} = V/I = V_H/I = R_H.$$

We now consider a Hall conductor of rectangular shape (Fig. 2). At boundaries *EF* and *GH* the tangential component of **E** is[6]

$$E_t = \frac{V_H}{b}\left(\tan\frac{\pi y}{2l}\right)^{\pm 2\delta/\pi} e^{\pm h(y)}, \quad \binom{+EF}{-GH}$$

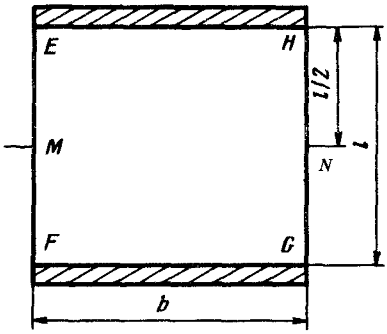

FIG. 2. Sample of rectangular shape with contacts on sides EH and FG. The Hall contacts are at points M and N.

$$h(y) = \sum_{n=0}^{\infty} \left[1 - \tanh\frac{(2n+1)\pi b}{l}\right] \cos\frac{(2n+1)\pi y}{l},$$

where $\tan\delta = \sigma_{\|}/\sigma_H$, and V_H is the potential difference across the Hall contacts M and N. Introducing $\eta = y/l$, we find the potential difference between points F, M and N, H to be

$$\Delta\varphi_{FM} = \Delta\varphi_{NH} = \frac{l}{b}\cos\delta \int_0^{\pi/2} (\tan\eta)^{2\delta/\pi} e^{-h(y)} d\eta.$$

The voltage drop of the sample is $V = V_H + 2\Delta\varphi_{FM}$.

If the sample is not too narrow, we would have $\pi b/l > 1$ and thus $h \cong 0$. Calculating[5] $\Delta\varphi_{FM}$ in first order in $\cos\delta$, we then find

$$R_{SD} = (V_H + 2\Delta\varphi)/I = R_H\left[1 + \cos\delta\left(\frac{l}{b} - \ln 2\right)\right].$$

In the evaluation of this integral, the points E and G, at which **E** is singular, are not in the range of integration. The arguments above make it a simple matter to carry out calculations for circuits including Hall and ordinary conductors (see Refs. 1 and 2, for example).

We wish to thank B. L. Al'tshuler and A. G. Aronov for useful discussions.

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Translated by Dave Parsons