Nuclear effects in deep inelastic scattering of leptons

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A correct calculation of the energy spectrum of the nucleus and of the Fermi motion of the nucleons can explain the EMC effect [Aubert *et al.*, Phys. Lett. **105B**, 403 (1983)] in terms of conventional nuclear physics, without the assumption of nucleon deformation and the existence of multiquark configurations in the nucleus.

We know that in the limit $Q^2 \to \infty$ the deep inelastic scattering cross section is proportional to the structure function $F_2(x)$ which corresponds to the target. Working from this proportionality and using the impulse approximation for scattering by a nucleus, we can represent the nucleon contribution to the structure function of the nucleus in terms of the structure function of the nucleon as follows:

$$AF_2^A(x) = \int d^4p S(p) F_2^N(x_p), \tag{1}$$

where $x = Q^2/2mq_0$, $x_p = Q^2/2pq$, $S(p)$ is the 4-momentum distribution of the nucleons in the nucleus (the spectral function)

$$S(p) = (2\pi)^{-3} \sum_\lambda |\varphi_\lambda(\mathbf{p})|^2 \delta(p_0 - m - \epsilon_\lambda) \tag{2}$$

$|\varphi_\lambda(\mathbf{p})|^2$ is the probability of finding a single nucleon with a momentum $\mathbf{p}$ in the nucleus and the probability of finding the rest of the nucleons in the $|(A-1)_\lambda\rangle$ state with an energy $E_\lambda^{A-1}, m + \epsilon_\lambda = M_A - E_\lambda^{A-1}$ [$M_A = Am(1+\mu/m)$ is the mass of the target nucleus, and μ is the binding energy per nucleon], and the summation is over all excited $|(A-1)_\lambda\rangle$ states. The $S(p)$ distribution, which is normalized to allow for the number of nucleons

$$\int d^4p\, S(p) = A, \tag{3}$$

can be found from the data on quasielastic knocking out of nucleons in the $(e,e'p)$ reactions.[3] The use of the energy spectrum of the intranuclear nucleons sets our approach apart from other studies,[4,5] in which it was assumed that the nucleons in the nucleons are found on the mass shell. To see what the consequences of this assumption might be, we will rewrite (1) in a convoluted form

$$AF_2^A(x) = \int dz f(z) F_2^N(x/z), \tag{4}$$

$$f(z) = \int d^4p\, S(p)\, \delta(z - pq/mq_0), \tag{5}$$

where $f(z)$ is the longitudinal-momentum distribution of the nucleons (in units of nucleons mass m). The bell-shaped function $f(z)$ has a maximum near the expectation value of z.

$$z_0 = \frac{1}{A} \int dz z f(z) = \frac{1}{A} \sum_\lambda \int \frac{d\mathbf{p}}{(2\pi)^3} |\varphi_\lambda(\mathbf{p})|^2 \left(1 + \frac{\epsilon_\lambda - \mathbf{pn}}{m}\right)$$

$$= 1 + \frac{\langle \epsilon_\lambda \rangle}{m}, \quad (\mathbf{n} = \mathbf{q}/q_0), \tag{6}$$

which can be expressed in terms of the mean separation energy of the nucleon $\langle \epsilon_\lambda \rangle$, and a width $\sim p_F/m$, where p_F is the momentum of the Fermi nucleons in the nucleus. In the region $x \ll 1$, the value of $\langle \epsilon_\lambda \rangle$ is all the information about the nucleus we need in order to determine $F_2^A(x)$. We can easily see this from the following expansion which can be derived from (4) by expanding $F_2^N(x/z)$ near $z = z_0$

$$F_2^A(x) = F_2^N\left(\frac{x}{z_0}\right) + \frac{1}{2}\left(\langle z^2 \rangle - z_0^2\right) \frac{\partial^2}{\partial z_0^2} F_2^N\left(\frac{x}{z_0}\right) + \ldots \cong F_2^N\left(\frac{x}{z_0}\right). \tag{7}$$

Using a familiar functional dependence $F_2^N(x)$, we find that (7) is valid in the region $x \lesssim 0.5$ and that it accounts for the decrease in the ratio $R(x) = F_2^A(x)/F_2^N(x)$ if $z_0 < 1$. We wish to emphasize that the condition $z_0 < 1$ is a necessary condition for any bound system ($\epsilon_\lambda < 0$). The degree of tilt of $R(x)$ in this region is determined by the quantity z_0. This quantity can be calculated by using the following relation which relates $\langle \epsilon_\lambda \rangle$ and the average kinetic energy of the nucleon in the nucleus, $\langle \mathbf{p}^2 \rangle / 2m$, in the case of two-body NN forces:[6]

$$\langle \epsilon_\lambda \rangle + \langle \mathbf{p}^2 \rangle / 2m = 2\mu. \tag{8}$$

Using in the case of an iron nucleus the estimate $\langle \mathbf{p}^2 \rangle / 2m = 3p_F^2/10m = 23$ MeV $p_F = 270$ MeV/c and $\mu = -8$ MeV, we find $\langle \epsilon_\lambda \rangle_{\text{Fe}} = -39$ MeV and $(z_0)_{\text{Fe}} = 0.96.$[1)]

In the region $x \gtrsim 0.5$ the parameter of expansion (7) is appreciable, and $f(z)$ becomes important. Figure 1 shows the results of calculations of the ratio $R(x)$ in the Fermi-gas model of the nucleons in an average potential[9] V. In this model we have

$$\varphi_\lambda(\mathbf{p}) = \delta(\mathbf{p} - \mathbf{p}')\, \theta(p_F - |\mathbf{p}|), \quad (\lambda = \mathbf{p}')$$

$$\epsilon_\lambda = V + \mathbf{p}^2/2m. \tag{9}$$

The potential was determined from the condition $\langle \epsilon_\lambda \rangle_{F\cdot g} = \langle \epsilon_\lambda \rangle_{\text{Fe}} (V = -62 \text{ MeV})$. In spite of its simplicity, this model represents rather well the main properties of the nucleus and, as we can see in the figure, describes the experiment satisfactorily in the

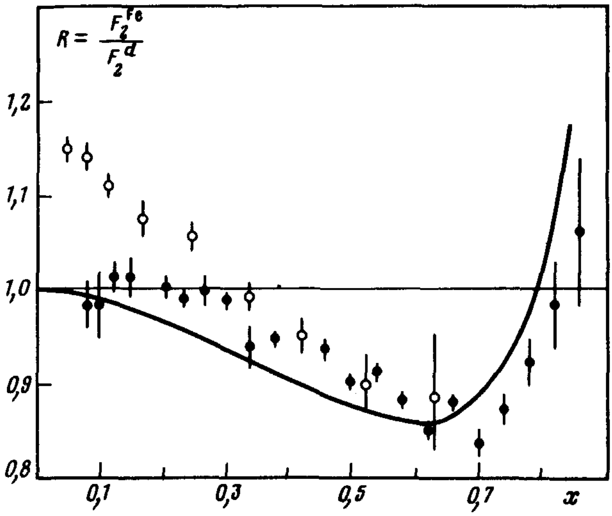

FIG. 1. Experimental data ○—EMC (Ref. 1); ●—SLAC (Ref. 2). In the calculations we used the parametrization of Ref. 10, $F_2^N(x) = 0.59\sqrt{x}(1-x)^{2.8} + 0.33\sqrt{x}(1-x)^{3.8} + 0.49(1-x)^8$.

region in which the data are noncontradictory.

We will now briefly consider the sum rules for F_2^A. From (1), (2), and (6) we find

$$\int_0^A dx F_2^A(x) = z_0 \langle x \rangle; \qquad \langle x \rangle = \int_0^1 dx F_2^N(x). \tag{10}$$

The presence of a factor $z_0 < M_A/Am$ in this equation is consistent with the energy conservation law. This mean that the nucleons account for only a part of the total energy of the nucleus. The remaining part of the energy is found in the fields that bind the nucleons in the nucleus. If we assume that mesons are such fields, then these mesons can be used to saturate (10). It is important to note that the mesonic contribution is concentrated[7,1] in the region $x < 0.2$ and has virtually no effect on the region $x > 0.2$ which is controlled by nucleons.

We can assert, therefore, that in the region $x > 0.2$ the EMC effect can be described satisfactorily in terms of the impulse approximation (provided that the energy spectrum of the nucleus is calculated correctly), without the assumption that the structure function of the nucleon changes in the nucleus. This circumstance, on the one hand leaves for the quark degrees of freedom in the nucleus less room than is ordinarily provided and, on the other, demonstrates the surprising stability of the nucleons in the nucleus.

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[1] We should point out in this connection that by using equations similar to (7) to fit the experimental data, Smith[7] and Garcia[8] have found that z_0 lies in the range 0.94–0.96.

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