Step potential of quarkonium

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(Submitted 4 June 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **42**, No. 3, 119–122 (10 August 1985)

It is suggested that a potential whose Lorentz-scalar part changes abruptly because of a breaking of chiral symmetry be used to describe quarkonium. This singularity is shown to be compatible with existing data on quarkonia and to lead to specific predictions for toponium.

The properties of quarkonia, consisting of a heavy quark and a heavy anti-quark, can be described well by a Schrödinger equation with a local static potential (Refs. 1 and 2, for example), which is usually chosen to be a smooth, monotonically increasing function which is independent of the flavor of the quarks.

A distinctive feature of the quark-quark interaction is that the quark confinement which it causes can be explained in a potential model only if the potential is described as a superposition of Lorentz-vector and Lorentz-scalar components. The behavior of the Lorentz-scalar part of the potential is linked[3] with a dynamic nature of the mass of a quark, which must change as the distance between quarks changes.

A change in the mass of light quarks is taken into account phenomenologically when one speaks in terms of "current" quarks with small masses and "constituent" quarks with comparatively large masses, which are manifested at short and long range, respectively. The success of models of both types indicates that there is a rather sharp transition from current quarks to constituent quarks (a rapid breaking of chiral symmetry). It has also been concluded from an analysis of the effective QCD Lagrangian[4] that the transition from current quarks to constituent quarks occurs very rapidly in a momentum interval on the order of 1 GeV (i.e., at distances of 0.1–0.2 fm).

The sharp change in the mass of a quark should lead to a steep feature (a "step" at $r = r_0$) in the potential in the Schrödinger equation in the potential approach. If the picture of an "overgrowth" by a gluon field is identical for light and heavy quarks, the height of the step in the potential must be equal to twice the difference between the masses of the constituent and current quarks, i.e., about 600 MeV. The position of this singularity along the scale of the distance r is chosen in the interval 0.1–0.2 fm specified above, by requiring that the calculated and experimental characteristics of bottomonium agree. The spectra and radiation widths of charmonium and bottomonium can be reproduced with a modified potential only if this potential is essentially the same as the earlier approximations[1,2] at distances greater than r_0. At shorter distances, however, it changes substantially, becoming flatter. In this way we can resolve the problem of the large value of the parameter Λ in all phenomenological potentials with asymptotic freedom:

$$V(q^2) \sim \frac{1}{q^2 \ln (q^2/\Lambda^2)} \quad \text{as} \quad q^2 \to \infty, \tag{1}$$

which has previously turned out to be on the order of 400–500 MeV, in contrast with the usually accepted value $\Lambda_{QCD} \simeq 100$ MeV. In a step potential, it would be essentially equal to Λ_{QCD}. This "flattening" of the potential results from the requirement that it describe the leptonic widths of the quarkonia, which are proportional to the average value of the derivative of the potential,

$$\Gamma_{ee} \sim \left\langle m \frac{\partial V}{\partial r} \right\rangle_{nS}, \tag{2}$$

since the only way to retain the values found previously[1] for Γ_{ee} while increasing the derivative of the potential in a certain region (because of the step) is to reduce it in another part of the space. The smooth behavior of the potential at short range also eliminates the difficulties[1] involving the reproduction of the hadronic decay widths of quarkonia, Γ_{had}, since a decrease in Λ leads to a decrease in the strong-interaction constant α_S [we reacall that $\alpha_S \sim 1/\ln(Q^2/\Lambda^2)$].

We will thus use the step potential shown in Fig. 1. It agrees completely with the potential used by us previously[1] at distances greater than r_0, has a discontinuity at $r = r_0$, and is distinguished by a small value $\Lambda = 100$ MeV at small distances. Shown for comparison in Fig. 1 are the potentials of Martin[5] and Ono.[6]

Table I shows properties calculated for bottomonium and toponium with this choice in potential. For the value $r_0 = 0.10$ fm, the characteristics (aside from the hadronic widths) of bottomonium are essentially the same as those found previously through the use of smooth potentials, and they agree quite well with the experimental characteristics (cf. the data in the reviews in Refs. 1 and 2). The calculated hadronic widths of bottomonium now agree well with experiment.

On the other hand, the predictions for several levels of toponium differ significantly from earlier predictions.[1] Since the mean square radius of the $1S$ state of toponium ($\langle r^2 \rangle_{1S}^{1/2} = 0.06$ fm, $m_t = 45$ GeV) is much smaller than r_0, the properties of this level (its mass and the leptonic and hadronic widths) are largely analogous to the properties of the $1S$ level of toponium in potentials corresponding to $\Lambda \simeq 300$ MeV. The $2S$ level lies near the jump and has some specific features. In the first place, the leptonic and hadronic widths of this level are large (with respect to those of the $1S$ level): $\Gamma(2S)/\Gamma(1S) = 0.4$–$0.5$ (for potentials with asymptotic freedom[1] it is ~ 0.2). The reason is that this level is "trapped" in a potential well of small radius r_0. The same circumstance explains the large level splitting, $M(2S) - M(1S) = 980$ MeV. More highly excited levels lie in a region in which the potentials with and without the step are

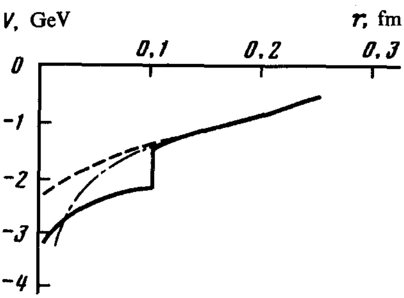

FIG. 1. Step potential of quarkonium (solid line) in comparison with the Martin potential[5] (dashed line) and the Ono potential[6] (dot-dashed line).

TABLE I. Properties of bottomonium and toponium in the potential model with the step potential shown in Fig. 1.

Bottomonium family

Characteristic	Expt.	Theo.	Characteristic	Expt.	Theo.
$M(1S)$, GeV	9,46	9,46	$\Gamma_{ee}(1S)$, keV	1,25	1,33
$M(2S)$, GeV	10,02	10,02	$\Gamma_{ee}(2S)$, keV	0,527	0,479
$M(3S)$, GeV	10,35	10,33	$\Gamma_{ee}(3S)$, keV	0,374	0,320
$M(4S)$, GeV	10,57	10,57	$\Gamma_{ee}(4S)$, keV	0,281	0,240
$M(1P)$, GeV	9,90	9,93	$\Gamma_{\rm had}(1S)$, keV	$26{,}6^{+7{,}8}_{-5{,}6}$	26,6
$M(2P)$, GeV	10,26	10,25	$\Gamma_{\rm had}(2S)$, keV	$12{,}7^{+5{,}8}_{-4{,}1}$	9,6
—	—	–	$\Gamma_{\rm had}(3S)$, keV	8 ± 2	6,5

Toponium family

Characteristic	Theo.	Characteristic	Theo.	Characteristic	Theo.
$M(1S)$, GeV	90,00	$\Gamma_{ee}(1S)$, keV	3,46	$\Gamma_{\rm had}(1S)$, keV	6,12
$M(2S)$, GeV	90,98	$\Gamma_{ee}(2S)/\Gamma_{ee}(1S)$	0,53	$\Gamma_{\rm had}(2S)$, keV	3,25
$M(3S)$, GeV	91,26	$\Gamma_{ee}(3S)/\Gamma_{ee}(1S)$	0,22	$\Gamma_{\rm had}(3S)$, keV	1,33
$M(1P)$, GeV	90,69	$\Gamma_{ee}(4S)/\Gamma_{ee}(1S)$	0,12	$\Gamma_{\rm had}(4S)$, keV	0,70
$M(2P)$, GeV	91,21	—	—	—	—

essentially coincident. The characteristics of these levels thus remain essentially the same in the case with a step. However, the step does lead to a pronounced change in the positions of the nodes and peaks of the wave functions and thus in the widths of the $E1$ and $M1$ transitions.

The introduction of a step potential for quarkonium, reflecting the dynamic nature of the quark mass, thus solves two problems which arise in a description of the properties of charmonium and bottomonium in the potential models: the problem of the discrepancy between theory and experiment in terms of the hadronic widths and the problem of the large value of the dimensional parameter Λ in the coupling constant. On the other hand, the agreement in terms of level positions and leptonic widths is retained.

The predictions for the $2S$ level of toponium differ significantly from all earlier predictions. If it turns out that experimental data on the $2S$ level of toponium put it far from the $1S$ level (980 MeV away—separated by a distance significantly greater than the separation of these levels in charmonium and bottomonium, $\sim$600 MeV), and if its leptonic and hadronic widths are smaller than the widths of the ground state by a factor of only about two, these results will indicate a rapid transition from the current quarks to the constituent quarks. The specific features of this transition can be determined by analyzing the numerous characteristics of the toponium family.

We wish to thank I. V. Andreev for useful discussions.

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Translated by Dave Parsons