Domain wall in a highly anisotropic ferromagnet

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Equations describing the nonlinear dynamics of a highly anisotropic magnetic material with spins $S = 1$ are used to analyze a domain wall which is unassociated with the flipping of the spins. The motion of this wall is accompanied by a wave of a rotation of the quadrupole moment.

The Landau-Lifshitz equation has recently been used to derive several important results in the nonlinear dynamics of magnetic materials.[1] On the other hand, as Landau and Lifshitz themselves have pointed out,[2] this approach is justified, strictly speaking, if the relativistic interactions are small in comparison with the exchange interactions. There are several examples, however, in which the single-ion magnetic anisotropy is comparable to or even significantly greater than the exchange. The static and dynamic properties of such crystals depend in an extremely strong way on the size of the spins; there are particularly marked differences between the properties of magnetic materials with integer and half-integer values of S (Refs. 3 and 4). It is thus hardly possible to construct an equation for highly anisotropic magnetic materials which would be as generally applicable a phenomenological equation as the Landau-Lifshitz equation. A self-consistent description of the nonlinear spin dynamics would have to be based on a study of the equations of motion for $4S(S+1)$ independent operators [e.g., the tensors O_{kq}, where $k = 1,...,2S$, which belong to the algebra $SU(2S+1)$].[1)]

In this letter we examine a typical situation which is in a sense the opposite of that in which one could take the semiclassical approach corresponding to the model of "rigid" spins, with $|\langle \mathbf{S}\rangle| = 1$. For a biaxial ferromagnet describable by a Hamiltonian

$$\mathcal{H} = -\sum_n \left[AS_{nz}^2 + B(S_{nx}^2 - S_{ny}^2)\right] - (I/2z)\sum_{nm} \mathbf{S}_n \mathbf{S}_m . \qquad (1)$$

with a sufficiently pronounced anisotropy $A - B < I$, a domain wall of the Bloch or Néel type would have a width smaller than the interatomic distance a, so that the semiclassical approximation would lead to an Ising domain wall. As we will show below, this model is unacceptably crude. For a self-consistent analysis of the dynamics, incorporating the quantum-mechanical nature of the spins, we use the equations of motion, of which we need two:

$$i\hbar\dot{S}_{nz} = [S_{nz}, \mathcal{H}] = -B(S_{n+}^2 - S_{n-}^2) - i(I/z)\sum_m (S_{ny}S_{mx} - S_{nx}S_{my}), \qquad (2)$$

$$i\hbar(\dot{S}_{n+}^2 - \dot{S}_{n-}^2) = -B\left[(8S(S+1) - 4)S_{nz} - 8S_{nz}^3\right] + 2(I/z)\sum_m [S_{n+}^2 + S_{n-}^2]S_{mz}$$
$$- (I/z)\sum_m \{[S_{n+}S_{nz} + S_{nz}S_{n+}]S_{m+} + [S_{n-}S_{nz} + S_{nz}S_{n-}]S_{m-}\}. \qquad (3)$$

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The last terms in (2) and (3) are related to excursions of the moments from the Z axis and are evidently not pertinent to the problem at hand. According to the known procedure,[5] it is easy to examine the exchange contribution to the effective Landau-Lifshitz field in these terms. An anisotropic contribution to this field arises upon splittings in the Hamiltonian of the type $S_{ni}S_{nj} \to S_{ni}\langle S_{nj}\rangle$, which make the system of equations for the linear spin operators S_{ni} closed under the condition $|\mathbf{S}_n| = S$.

Taking an average of (2) and (3) over the Hartree wave function

$$\Psi_0 = \prod_n \{ e^{-i\gamma_n}\cos\varphi_n\,|1> + e^{i\gamma_n}\sin\varphi_n\,|-1> \},$$

noting that we have $S^3_{nz} = S_{nz}$ in the case $S = 1$, and introducing $\sigma_n = \langle S_{nz}\rangle_0 = \cos 2\varphi_n$, $Q_n = \sqrt{1-\sigma_n^2}$, we find the equations which we need to describe the dynamics of the spins of a highly anisotropic magnetic material, (1):

$$\hbar\dot{\sigma}_n = -2BQ_n\sin 2\gamma_n\,, \tag{4a}$$

$$\hbar\dot{\gamma}_n = B(\sigma_n/Q_n)\cos 2\gamma_n - (I/z)\sum_m \sigma_m\,. \tag{4b}$$

Differentiating (4a), and eliminating $\dot{\gamma}_n$, and γ_n from (4a) and (4b), we find a second-order differential-difference equation for $\sigma_n(t)$. We seek a solution of the equation for $\sigma(X,t)$ which corresponds to this equation in the continuum approximation in the form $\sigma = \sigma(\xi)$ under the boundary conditions $\sigma(\infty) = -\sigma(-\infty) = s_0$, $\sigma'(\pm\infty) = 0$, where $\xi = (X - Vt)\sqrt{2}/a$, $s_0 = \sqrt{1-b^2}$ is the solution for the homogeneous state, and $b = B/I$. Assuming $\cos 2\gamma = 1 - (u^2/2b)(\sigma'/Q)^2$, $(u = V\hbar/\sqrt{2a^2BI})$ in the same approximation, we find the first integral

$$(1 + u^2/2b)(\sigma')^2 = (Q-b)^2 + \frac{3u^2}{1+u^2/b}(Q-b) - \frac{3b(b-u^2)}{1+u^2/b}\left[1 - \left(\frac{b-u^2}{Q-u^2}\right)^{u^2/b}\right]. \tag{5}$$

We can draw some preliminary conclusions from (5). The limiting velocity at which a plane domain wall can move without being destroyed is $u_k = \sqrt{b}$: At $u \geqslant u_k$, only periodic solutions with an amplitude $|\sigma| \leqslant \sqrt{1-u^4}$. are permissible. The width of the domain wall in the case $u = 0$ is $\delta_0 = 2a/\sqrt{1-b}$, so that the condition for the applicability of the continuum description is the inequality $1 - b \ll 1$. In the limit $u \to u_k$ we have $\delta \to \delta_k = a\sqrt{2}$. The final solution is, within terms of up to second order in $(u/u_k)^2$,

$$\frac{\xi}{\sqrt{1-(u/u_k)^2}} = \frac{b}{s_0}\,\mathrm{Arsh}\left(\frac{s_0\sigma}{\sqrt{1-\sigma^2} - b}\right) + \arcsin(\sigma) + o(u^2/u_k^2), \tag{6}$$

$$\sin 2\gamma = \frac{u}{u_k}\frac{\sigma'}{\sqrt{1-\sigma^2}} \approx \frac{u}{u_k}\frac{s_0^2}{1 + b\,\mathrm{ch}(s_0\xi)}\,. \tag{7}$$

This solution describes a moving domain wall [Eq. (6)] and a localized wave of a rotation of the quadrupole moment through an angle $\gamma(\xi)$ in the xy plane which accompanies the moving domain wall [Eq. (7)]. [In the case $u=0$, solution (6) is the same as the exact solution of the self-consistent problem, which can be found from the inhomogeneous version of the equation used in Ref. 3 to describe a continuous metamagnetic transition in an antiferromagnet with spins $S=1$.]

The energy corresponding to solution (6), (7) is conveniently written in the form $E=E_0+mV^2/2$, where

$$E_0 = I(\arcsin(s_0) - bs_0)/\sqrt{2} \approx \frac{\sqrt{2}}{3} Is_0^3, \quad m \approx \frac{\sqrt{2}\hbar^2 Is_0^3}{6\,aB} \tag{8}$$

are the energy and inertial mass, respectively, of the fixed domain wall.

If the condition $1-b \ll 1$ (under which we can take the limit of a continuum description) does not hold, we must solve system (4a), (4b) directly. For a fixed domain wall, two different solutions are possible here. One solution (a) is symmetric with respect to a plane which coincides with an atomic plane, while the other (b) is symmetric with respect to a plane which lies halfway between two atomic planes:

a) $\sigma_0=0,\quad \sigma_1=-\sigma_{-1}=s_0(1-3b^2/2),\ \sigma_2=-\sigma_{-2}=s_0-O(b^4), \ldots, E_0^a=1-b;$

b) $\sigma_1=-\sigma_0=s_0-\beta[1-\beta/2+o(\beta^2)], \sigma_2=-\sigma_{-1}=s_0-o(b^{8/3}), \ldots,\ E_0^b=1-3\beta^2/2,$

where $\beta=(2b^2)^{1/3}$. It is not difficult to see that under the condition $b<2/27$ a domain wall of type a) is favored. The difference $E_0^a-E_0^b$ may be thought of as an additional periodic potential in which the domain wall is moving (this potential may be pertinent at high velocities and in the case $b \lesssim 1$, in which we have $\delta \to a\sqrt{2}$).

We wish to emphasize that the type of domain wall considered in this paper is of an especially quantum-mechanical nature and could arise only at integer values of S. For half-integer values of S, there are no grounds of any sort for ignoring the transverse components of the spins, since the quantity $\langle \mathbf{S}_n \rangle_0$ does not vanish, although it may decrease substantially in a wall. For "classical" spins, a domain wall with a magnetization of variable magnitude, without a rotation of the magnetization (on the average), is possible only as a substantially statistical effect in the case $T_c - T \ll T_c$ (Refs. 6 and 7).

[1)]The problem of a complete description of a spin system was discussed in Ref. 8, where a closed system of nonlinear equations was also derived for the case $S=1$.

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