Effect of Bose condensation on inelastic processes in gases

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A change in the structure of the wave function of the initial state causes the probability for an inelastic process in a Bose gas to decrease sharply upon the appearance of a Bose condensate. In a gas of spin-polarized atomic hydrogen, the result is a decrease in the rate of three-body dipole recombination. There is accordingly the possibility of observing a phase transition.

1. The presence of a Bose condensate should generally cause a substantial change in the probabilities for inelastic processes which occur in a system of Bose particles. Simple arguments make this assertion easy to understand. We consider a separate elementary process which involves several identical particles. We need to give the wave function the appropriate symmetry. If the particles are in a condensate, this step is not necessary, so that the transition amplitude changes.

This phenomenon turns out to be very important for a system such as spin-polarized atomic hydrogen (H↓), which remains a gas down to $T=0$. This system, which is metastable, decays as a result of inelastic depolarization and recombination processes, which were studied in detail in Ref. 1 for the case without a condensate. As has been predicted theoretically[1,2] and observed experimentally,[3–5] in an H↓ gas there is an inescapable decay mechanism which does not require a threshold. It involves a three-body recombination through a virtual change in the spin configuration due to a dipole-dipole interaction. As the temperature T decreases (but remains above the Bose condensation temperature T_c), the probability for this process becomes completely independent of T. As we will see below, the probability for three-body recombination decreases by a factor of six in a Bose condensate and thereby reduces the decay rate.

Perhaps the most important point is that here it becomes possible to detect the appearance of a Bose condensate and to measure its density, from the decrease in the recombination rate with decreasing T. This point is even more important in that the difficulty of reaching the region of Bose condensation may dictate a very restrictive experimental geometry (see Refs. 6 and 7, for example).

In this paper we directly analyze the effect of a Bose condensate on the processes that occur in the volume. A similar decrease in the rate of inelastic processes also occurs, however, in a two-dimensional gas phase which forms during adsorption on a surface. At $T=0$, this point is obvious, since an ordinary Bose condensate forms. Even at a nonzero temperature, however, at which there is no condensate in the ordinary sense, the presence of a "quasicondensate" (a condensate with a fluctuating phase) at a temperature below the point of the phase transitions leads to qualitatively the same results (a detailed analysis will be published separately). This circumstance is very important for the system of spin-polarized hydrogen, since three-body dipole

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recombination in a surface phase (adsorbed on helium) is in many cases the dominant mechanism for the decay of the entire H↓ phase.[1,2,4,5,7] Here there is the possibility of detecting the phase transition in a two-dimensional system also; the conditions for this detection are much easier to arrange.

2. Let us examine the three-body dipole recombination which occurs in an H↓ gas and which gives rise to the formation of an H_2^* molecule and an H atom with kinetic energies large in comparison with T. Denoting by $\hat{H}'$ the Hamiltonian of the interaction corresponding to this transition, and assuming the recombination rate to be low, we find the following expression for the number of transitions per unit time ($\hbar = 1$):

$$W = 2\pi \sum_{i,f} \rho_i \,|\langle f|\hat{H}'|i\rangle|^2\, \delta(E_f - E_i) = \int_{-\infty}^{\infty} dt \sum_i \rho_i \langle i|\hat{H}'(0)\hat{H}'(t)|i\rangle , \quad (1)$$

where

$$\hat{H}'(t) = e^{i\hat{H}_0 t}\hat{H}' e^{-i\hat{H}_0 t},$$

$\hat{H}_0$ is the Hamiltonian of the system when $\hat{H}'$ is ignored, and ρ_i is the equilibrium density matrix.

An expression for the amplitude of this three-body process was derived in the gas approximation, $nR_0^3 \ll 1$, in Ref. 1 (R_0 is the effective radius of the interatomic interaction, and n is the particle density). At low temperatures, where the thermal momentum k_T satisfies $k_T R_0 \ll 1$, this amplitude becomes completely independent of the initial momenta of the particles. Under these conditions, for a particular transition pathway corresponding to a definite excited state of the molecule and a definite polarization of the third body, the vertex in $\hat{H}'$ can be replaced by a constant value $\tilde{V}$ corresponding to this amplitude, and the interaction itself becomes a point interaction. Under the assumption that all the particles are in a common spin state, the Hamiltonian $\hat{H}$ can be written in second quantization as

$$\hat{H}' = \tilde{V} \int d\mathbf{r}\, \hat{\varphi}^+(\mathbf{r})\, \hat{\Psi}^+(\mathbf{r})\, \hat{\Psi}(\mathbf{r})\, \hat{\Psi}(\mathbf{r})\, \hat{\Psi}(\mathbf{r}) + \text{H.a.} \quad (2)$$

Here $\hat{\Psi}(\mathbf{r})$ and $\hat{\varphi}(\mathbf{r})$ are the $\hat{\Psi}$ operators for H atoms and H_2 molecules, respectively. Treating the motion of the excited molecule and the fast atom in the final state as a free motion with energies $(k_1^2/4m) - E_0$ and $k_2^2/2m$, respectively (m is the mass of the atom, and E_0 is the binding energy of the excited molecule), and noting that these states are not populated, we can rewrite the correlation function in (1) as

$$\langle i|\hat{H}'(0)\hat{H}'(t)|i\rangle = |\tilde{V}|^2 \sum_{\mathbf{k}_1\mathbf{k}_2} \int d\mathbf{r}_1 d\mathbf{r}_2 \langle i|\hat{\Psi}^+(\mathbf{r}_2,0)\,\hat{\Psi}^+(\mathbf{r}_2,0)\,\hat{\Psi}^+(\mathbf{r}_2,0)$$
$$\times \hat{\Psi}(\mathbf{r}_1,t)\,\hat{\Psi}(\mathbf{r}_1,t)\,\hat{\Psi}(\mathbf{r}_1,t)|i\rangle\, e^{i(\mathbf{k}_1+\mathbf{k}_2)(\mathbf{r}_1-\mathbf{r}_2)}\, e^{i\left(\frac{k_2^2}{2m}+\frac{k_1^2}{4m}-E_0\right)t} . \quad (3)$$

The scale dimension over which the correlation function in (3) varies corresponds to an ordinary correlation length in a Bose gas, which, in the limit $T \to 0$, becomes $(na)^{-1/2}$, where a is the scattering length. At nonzero values of T, it becomes equal to the thermal de Broglie wavelength λ_T. An integration over the momenta k_1 and k_2 in (3), whose scale is dictated by E_0, causes distances $|\mathbf{r}_1 - \mathbf{r}_2|$ far smaller than the correlation lengths to become effective. We can thus set $\mathbf{r}_1 = \mathbf{r}_2$ in the correlation function. Carrying out the integration over $\mathbf{r}_1$ and $\mathbf{r}_2$ explicitly, and returning to the original expression, (1), we find

$$W = 6|\tilde{V}|^2 n^3 \sum_{\mathbf{k}} \int_{-\infty}^{\infty} dt K(t) e^{i\left(\frac{3k^2}{4m} - E_0\right)t} ; \tag{4}$$

$$K(t) = \frac{1}{6n^3} \sum_i \rho_i < i | \hat{\Psi}^+(0,0) \hat{\Psi}^+(0,0) \hat{\Psi}^+(0,0) \hat{\Psi}(0,t) \hat{\Psi}(0,t) \hat{\Psi}(0,t) | i > . \tag{5}$$

The integral in (4) is dominated by times $t \sim (1/E_0)$. This time scale is small in comparison with the times $(4\pi an/m)^{-1}$ and $1/T$, which are characteristic times for the time-dependent correlations in an interacting Bose gase. It follows immediately that our problem is actually determined by the correlation function $K(0)$. Here

$$W = W_0 K(0) ;$$

$$W_0 = 12\pi |\tilde{V}|^2 n^3 \sum_{\mathbf{k}} \delta\left(\frac{3k^2}{4m} - E_0\right) . \tag{6}$$

The probability W_0 becomes the same as that found for three-body recombination in Ref. 1 if it is summed over the possible transition pathways. The correlation function $K(0)$ remains the same for all pathways.

The structure of results (5) and (6) is the same for three-body inelastic processes of any sort, provided that the kinetic energy of the particles in the final state is large in comparison with T.

3. Let us examine the temperature dependence of $K(0)$ at $T < T_c$. For this purpose, we write the operator Ψ in the standard form (see Ref. 8, for example)

$$\hat{\Psi} = \Psi_0 + \hat{\Psi}' ,$$

where Ψ_0 is the condensate wave function, and $\hat{\Psi}'$ refers to only the particles which are above the condensate. Substituting this expression into (5), and calculating the correlation function in the approximation of an ideal gas, we find

$$K(0) = \frac{1}{6n^3} [n_0^3 + 9n_0^2 n' + 18 n_0 n'^2 + 6n'^3] . \tag{7}$$

Here $n_0 = |\Psi_0|^2$ is the condensate density, $n'(T) = \langle \hat{\Psi}'^+(0,0)\hat{\Psi}'(0,0) \rangle$ is the density of particles above the condensate, and $n_0 + n' = n$. Expression (7) also applies at $T > T_c$ if we note that we have $n_0 = 0$ in this region. Here $K(0) = 1$. If $T = 0$, we have $n' = 0$ and

$$K(0) = 1/6 . \tag{8}$$

When we consider a Bose gas which is slightly nonideal, we can use a Bogolyubov transformation[9] and switch from the particle absorption (creation) operators $\hat{a}_{\mathbf{k}}$ in $\hat{\Psi}'(0,0) = \Sigma_{\mathbf{k}\neq 0} \hat{a}_{\mathbf{k}}$ to quasiparticle operators $\hat{\mathbf{b}}_{\mathbf{k}}$, $\mathbf{b}_{\mathbf{k}}^+$ and, correspondingly, to a new vacuum. The correlation function in (5) can then again be calculated directly. At $T = 0$ we have

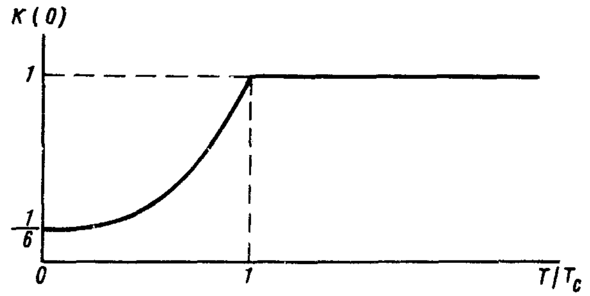

FIG. 1.

$$K(0) = \frac{1}{6}\left(1 + \frac{64}{\sqrt{\pi}}\sqrt{na^3}\right), \tag{9}$$

corresponding to the first correction in the gas parameter. It should be noted that in calculated $K(0)$ we are now forced to consider the renormalization of the vertex in $\hat{H}'$ in (2) in order to eliminate the formal divergence at large momenta which arises when anomalous expectation values of the type $\langle\Psi'\Psi'\rangle$ are taken into account. (This procedure is analogous to the standard procedure in the theory of a weakly interacting Bose gas; see Ref. 8, for example.)

Results (8) and (9) show that the rate of three-body inelastic processes falls off by a factor of about six in the condensate. As the temperature is lowered, the rate thus initially approaches a constant value at $k_T R_0 \ll 1$; then, at $T < T_c$, it begins to decrease sharply. Over nearly the entire temperature range the T dependence of $K(0)$ is determined by expression (7), with the value $n' = n(T/T_c)^{3/2}$, which is characteristic of an ideal gas (Fig. 1). The only deviation from this behavior occurs at very low temperatures, $T \sim (an/m)$, where the number of particles above the condensate due to thermal excitation and the number due to the interaction of particles are comparable in magnitude.

The appearance of a sharp temperature dependence of the rate of three-body recombination at $T < T_c$ opens up some unique experimental opportunities for detecting a phase transition accompanied by the formation of a Bose condensate in an H↓ gas.

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