Gell-Mann–Low function in supersymmetric electrodynamics

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The exact β function is derived in supersymmetric electrodynamics. The relationship between the second-order and higher-order coefficients and the infrared regularization is analyzed.

An expansion in the coupling constant in supersymmetry models[1] has certain "extraordinary" properties. For the F terms, for example, there are no corrections at all,[2] and for the β function these corrections are exhausted by the first loop in the $N=2$ gauge theory.[3] Even more surprising are those cases in which the perturbation-theory series is not truncated but can be calculated exactly. The first example of this type is the β function in supersymmetric gluodynamics, which was calculated in Ref. 4. That study was based on an analysis of the one-instanton amplitude. If this amplitude is known exactly in terms of the seed parameters (and this is the case[4]), the renormalizability of the theory fixes the exact β function. For the SU(N_c) gauge group, for example, we have

$$\beta(\alpha_s) = -\frac{\alpha_s^2}{2\pi}\,\frac{3N_c}{[1-(N_c\alpha_s/2\pi)]}\,. \tag{1}$$

A crucial point is that there are no diagrams with two, three, etc., loops in the instanton background field.[4] It was later recognized that this fact is only a particular case of a general situation. Specifically, if we examine the supersymmetric gauge theory for $d=4$, regularized in both the ultraviolet and infrared regions (for $d=4$), a calculation of the effective action in an external field reduces to a single loop. An assertion regarding a single loop is contained (in the form of a question) in the final paragraph in Ref. 5. This particular problem—the relationship between the absence of higher-order loops in the effective action and the infrared regularization—was the subject of Ref. 6.

In the present paper we discuss this idea in connection with practical applications. We derive the following exact relation in supersymmetric electrodynamics:

$$\beta(\alpha) = \frac{\alpha^2}{\pi}\,(1+\gamma_m), \tag{2}$$

where β is the Gell-Mann–Low function, $\beta(\alpha_0) = d\alpha_0/d\ln M_0$, and γ_m is the anomalous dimensionality of the mass (of the electron), given by

$$\gamma_m = -\,d\ln m_0/d\ln M_0\,,$$

where M_0 is the ultraviolet cutoff, and the subscript 0 specifies the bare quantity. In the single-loop approximation we have $\gamma_m = \alpha/\pi + \cdots$, and thus

$$\beta^{(2)}/\beta^{(1)} = \alpha/\pi, \tag{4}$$

in agreement with standard calculations[1] ($\beta^{(i)}$ is the i-th coefficient in the β function). From the k-loop coefficient in γ_m we find the $(k+1)$- loop coefficient in β. If the SUSY is not violated, the anomalous dimensionality of the mass differs only in sign from the anomalous dimensionality of the matter field: $\gamma_{(\psi)} = d\ln Z/d\ln M_0 = -\gamma_m$.

Although there are no instantons in supersymmetric electrodynamics, the general logic of the derivation of (2) is the same as that in Ref. 4. Specifically, the theory is infrared-regularized by the introduction of a mass of matter. We then find an expression for the physical charge in the momentum region below the mass in terms of the seed parameters. This expression reduces to a single-loop expression. We then require that the exclusive dependence on the regulator mass M_0 be cancelled by an implicit dependence which enters through the bare quantities α_0 and m_0 (the bare mass of the electron). In this manner we find (2).

A distinction from Ref. 4 is that in Ref. 4 the infrared regularization is achieved by means of a background instanton field, in terms of which no expansion is carried out. In supersymmetric electrodynamics we expand in the background field, and the infrared regularization is achieved by introducing a mass in the matter fields.

We recall that the initial action in supersymmetric electrodynamics (SUED) is

$$S_{\text{SUED}} = \frac{1}{4e_0^2}\int d^4x d^2\theta W^2 + \frac{1}{4}\int d^4x d^2\theta d^2\bar{\theta}(\bar{T}e^V T + \bar{U}e^{-V}U) + \left(\frac{m_0}{2}\int d^4x d^2\theta TU + \text{H.a.}\right), \tag{5}$$

where T and U are two chiral superfields with charges $+1$ and -1, respectively, and m_0 is the bare mass. The four-dimensional ultraviolet regularization which conserves SUSY is achieved by introducing Pauli-Willars fields $T^{(i)}$, $U^{(i)}$ with the masses M_i for the matter plus higher-order derivatives for the gauge field V. We then have a propagator which descends well in momentum space.

The renormalized charge e^2 is defined as the coefficient of W^2 in the effective action $S_{\text{eff}} = (1/4e^2)\int d^4x d^2\theta W^2$. The coefficient in front of this construction is conveniently calculated from superdiagrams in the external-field method,[8] which guarantees supergauge invariance in the external field. In general, the loop contributions to the effective action reduce to

$$\int d^4x d^2\theta d^2\bar{\theta} f(x,\theta,\bar{\theta}), \tag{6}$$

where f is a function which has supergauge invariance with respect to the external field. A simple dimensionality analysis then shows that (under the condition of infrared regularization) structures of the type in (5) do not arise, and we have $\Delta S_{\rm eff} = 0$. The only exceptional case is a single-loop diagram with an internal chiral superfield.[4–6] The contribution from such a diagram does not reduce to the form in (6) and does not vanish.

The *exact* expression for the effective action in supersymmetric electrodynamics in the low-energy limit, $p \ll m$, is thus

$$S_{\rm eff} = \frac{1}{4}\left(\frac{1}{e_0^2} + \frac{1}{8\pi^2}\ln\frac{M_0^2}{m_0^2}\right)\int d^4x d^2\theta W^2 + \text{H.a.} \tag{7}$$

(the loop was calculated with the chiral superfields T and U; this approach corresponds to an electron and two charged scalar particles). We thus have

$$\frac{1}{\alpha} = \frac{1}{\alpha_0} + \frac{1}{2\pi}\ln\frac{M_0^2}{m_0^2}\,. \tag{8}$$

The renormalized charge α and the renormalized mass m should not depend on M_0; this circumstance determines the implicit M_0 dependence of the bare quantities α_0 and m_0. Differentiating (8) with respect to $\ln M_0$, we find (2).

The ultraviolet contribution proper to the charge renormalization stems from the differentiation of $\ln M_0^2$ on the right side of (8) and is exhausted by *a single loop.* Nevertheless, the β function in (2) is not a single-loop function; it instead contains all orders. The second and higher-order loops stem from the differentiation of $\ln M_0^2$ with respect to $\ln M_0$. We thus see that the calculation of the β function depends strongly on the infrared redefinition. It is interesting in this connection to compare result (2) with a direct calculation of two-loop diagrams in component form.[7]

Using the Fock-Schwinger gauge for the external photon field (this technique is reviewed in Ref. 9), we find that the problem is trivialized. The term $\ln M_0^2 F^2_{\mu\nu}$ in $S_{\rm eff}$ stems from the diagrams (Fig. 1) in which the solid lines represent propagators of charged particles in the external field. With $m_0 = 0$, i.e., if the momenta of the external field satisfy $q \gg m$, the calculation of these diagrams gives us a quantity which is by no means zero and which corresponds to the value of $\beta^{(2)}$ [see (4)]. However, in the case $q \ll m$, i.e., when a contribution from distances $x \sim m^{-1}$ is added, the two-loop contribution vanishes, in accordance with (7). This result means that the effect acquired at small distances cancels out exactly with the effect from large distances.

Finally, we can compare (2) with the expression for the β function [Eq. (2) of Ref. 6] derived in a non-Abelian gauge theory with matter by means of instanton calculus. If we discard from that expression the part which does not depend on the presence of the matter fields (i.e., if we set $n_v = n_\lambda = 0$), and if we take into account the difference

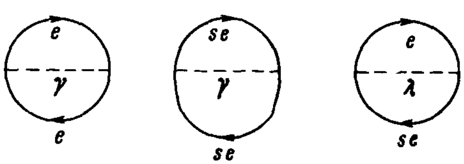

FIG. 1.

in the definitions of γ and γ_m and in the renormalization of the charges, we again reproduce (2).

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[1)]A component calculation of the two-loop β function in supersymmetric electrodynamics must be given somewhere in the literature, but we have not been able to find it. We have verified result (4), developed for this method, which is extremely simple and effective both in supersymmetry theories and ordinary theories. This method is of interest in its own right.[7] In principle, $\beta^{(2)}$ can also be extracted from Ref. 5, where a superdiagram *technique* is used.

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