Current distribution in a 2D electron layer with a quantized Hall resistance

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A semiempirical distribution of the current along the sample is calculated for conditions corresponding to the quantum Hall effect. This distribution explains experimental results. Near the center of the plateau, the current is concentrated in a narrow region which moves from one bank of the sample to the other upon a change in concentration.

The distribution of the current density j_x along the cross section of the sample (in the y direction) under the conditions of the quantum Hall effect had not been determined until recently.[1] An experiment carried out recently by Ebert, von Klitzing, and Weimann[2] revealed a current distribution which was unexpected at first glance: When the coefficient (ν) describing the filling of the Landau levels is far from an integer value, the current flows uniformly along the cross section of the sample. As ν approaches an integer value, i.e., the center of a plateau, the conducting region of the sample contracts into a filament ($\sim$0.6 mm wide in a 5-mm-wide sample). This narrow region moves from one bank of the sample to the other in the interval $\Delta\nu \lesssim 0.2$ ($\nu = n_s/n_H$, where n_s is the surface density of electrons, and n_H is the density of sites at the Landau level). The current distribution in the sample in those experiments was highly dependent on the concentration gradient along the y direction (i.e., transverse with respect to the current).

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In the present letter we show that a nucleating uniform current distribution is unstable under the conditions of the quantum Hall effect. In the next approximation, on the other hand, it follows from this distribution that the current "contracts" into a filament that moves along the y direction. The scale width of this filament is determined by the distance $(d\sigma_{xx}/dy)^{-1}\sigma_{xx}$, which is the distance over which there is a significant change in the component σ_{xx} of the conductivity tensor due to the concentration gradient.

System of equations. The following expression holds in the region of the plateau in ρ_{xy}:

$$j_x(y) = [\sigma_{xy}^2 / \sigma_{xx}(y)] E_x . \quad (1)$$

We are interested in the change in $j_x(y)$ only over large distances, $dy \gg L$, where $L \sim 10^{-5}$ cm is a scale length of the potential fluctuations. We assume that the carrier density distribution is in a steady state, so that we have $j_y = 0$. Applying the condition curl $E = 0$, we find $e_x(y) = \text{const}$. Since we have $\sigma_{xy} = \text{const} = (25\ 812/\nu)^{-1}$ S, we find from (1) that the current flows predominantly in those parts of the sample where σ_{xx} is small. A similar result was predicted earlier[3] on the basis of qualitative considerations. We express the field E_x in terms of the total current flowing through the sample, J_x:

$$E_x = J_x \sigma_{xy}^{-2} \left[\int_0^W \sigma_{xx}^{-1}\, dy \right]^{-1} . \quad (2)$$

To find $\sigma_{xx}(y)$, we use (first) the empirical linear relationship[4] between σ_{xx} and a small deviation δv_{xy}: $\sigma_{xx} \simeq \delta\sigma_{xy}/0.2$. Second, we use the phenomenological theory of Ref. 4, which has $\delta\sigma_{xy}/\sigma_{xy}$ equal to the relative change in the density of delocalized electrons. Since the delocalized states are concentrated along the energy (ϵ) scale near the center of Landau levels, ϵ_{j0}, we can write

$$\sigma_{xx}(y) = \left(\frac{e^2}{h}\right) \frac{1}{0,2} \{1 - f_F(\epsilon_{i,0}, \epsilon(y)) + f_F(\epsilon_{i,0} + \Delta\epsilon, \epsilon_F(y))\} . \quad (3)$$

Here we are considering only the two nearest Landau levels (which are separated by an energy gap $\Delta\epsilon$), between the Fermi level lies (Fig. 1); and $f_F(\epsilon, \epsilon_F)$ is the Fermi

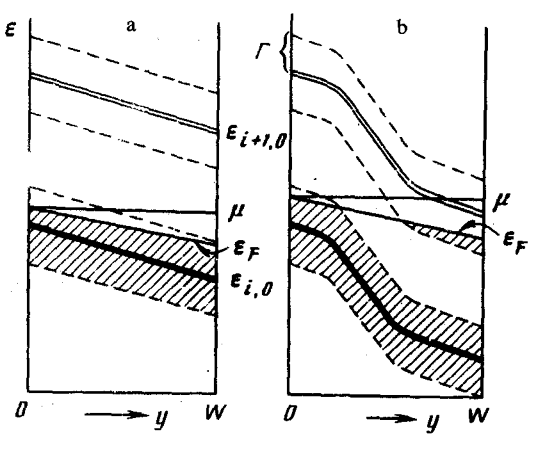

FIG. 1. Schematic diagram of the changes in the energy of the Landau levels and in the Fermi energy along the cross section of a metal-insulator-semiconductor structure during current flow. a—Far from the center of the ρ_{xy} plateau; b—at the center of the ρ_{xy} plateau.

distribution function.

The value of $\epsilon_F(0)$ is found from the integral equation

$$\nu = \sum_l \int_0^\infty D_l(\epsilon) f_F(\epsilon, \epsilon_F(0)) d\epsilon . \tag{4}$$

To simplify the calculation, we choose $D(\epsilon)$ to be a Gaussian function: $D(\epsilon) = (\sqrt{\pi}\Gamma)^{-1} \times \exp\{-(\epsilon - \epsilon_{i,0})/\Gamma]^2\}$. The Fermi energy can be written as a function of the coordinate y as follows:

$$\epsilon_F(y) = \epsilon(y = 0) + \int_0^y (d\epsilon_F / dy) dy . \tag{5}$$

In this expression we have assumed, for definiteness, that the density, the Fermi energy, or the gate voltage is given with respect to one of the contacts (the drain or the source). One of the banks of the sample (e.g., the left) has a potential equal to the potential of this contact, and we place the origin of coordinates at this bank ($y = 0$).

In the experiments of Ref. 2, the density gradient ($\Delta n_s / n_s \sim 2\%$ along the width of the sample) was rigidly fixed; in a metal-insulator-semiconductor structure, a gradient arises automatically[4] during a current flow:

$$d\epsilon_F / dy = j_x(y) / [K D(\epsilon) \sigma_{xy}] . \tag{6}$$

Here the coefficient K ($\sim 10^{11}$ cm$^{-2}\cdot$V^{-1}) is a measure of the capacitance of the metal-insulator-semiconductor structure.

Equations (1)–(6) can be used to find the solution which we are seeking, $j_x(y)$. Let us first determine the qualitative behavior of this solution. Figure 1 is a sketch of the level energy and ϵ_F along the y coordinate[1)] found under the assumption of a uniform nucleating current distribution. Far from the center of the plateau, where $\epsilon_F(y)$ lies entirely in a single wing of $D_i(\epsilon)$ (Fig. 1a), σ_{xx} depends only weakly on y, and the current distribution should not be greatly different from uniform. Near the center of the plateau, in contrast, or at high density gradients, a uniform distribution is no longer a solution of the problem. When the line $\epsilon_F(y)$ intersects the region of the energy gap (Fig. 1b), $\sigma_{xx}(y)$ changes dramatically. In those regions of the sample where $\epsilon_F(y)$ is in the gap, i.e., at a maximum distance from delocalized states, σ_{xx} is at a minimum. We would thus expect that it would be in these regions that most of the current would flow. As the density is varied, this region moves from one bank of the sample to the other until the line of $\epsilon_F(y)$ again lies entirely within the wing $D_{i+1}(\epsilon)$.

Figure 2 shows a family of y profiles of the current for various ν near $\nu = 4$ according to calculations from Eqs. (1)–(6) by a method of successive approximations for $J_x = 10\ \mu$A (the change in the density is 0.6% along the width of the sample). As a zeroth approximation we specify a uniform current distribution, $j_x = J_x / W$, and thus a constant density gradient. Shown at the left in Fig. 2 by the solid line is the fraction of the current which flows through cross section a–b, of width $W/2$. The dashed line shows the result derived in the zeroth approximation; it corresponds to a rigidly fixed constant density gradient of width $2\%/W$. The leftward shift of the peaks from $\nu = 4$ is attributed to our choice of an origin for the density scale at

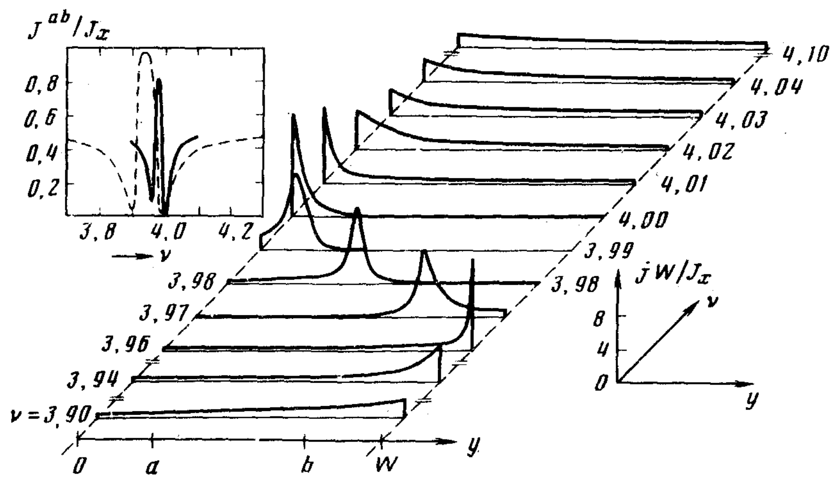

FIG. 2. Profile of the current distribution along the cross section of the sample for various values of the density near $\nu = 4$. The coordinate system and the scale along the vertical axis are shown at the right. Shown at the left is the fraction of the current that flows in cross section *a–b*, as a function of ν. Solid line—For a metal-insulator-semiconductor structure with $J_x = 10\ \mu$A; dashed line—with a rigidly fixed density gradient of $2\%/W$. The parameter values are $n_H = 2\times 10^{11}$ cm^{-2} ($H = 80$ kOe), $T = 1$ K, $\Delta\epsilon = 30$ K, and $\Gamma = 7$ K (Ref. 5).

the point $y = 0$. As the density gradient (or the current J_x) increases, the current filament contracts, and it moves slowly along the width of the sample. The shape of the peak $J^{ab}(\nu)$ depends very strongly on the state density in the region of the energy gap and can be used to find $D(\epsilon)$.

In summary, these results furnish a qualitative explanation for the experimental data of Ref. 2 and predict an analogous current distribution in a metal-insulator-semiconductor structure even if the density gradient is not specified externally.

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[1)]In contrast with Ref. 4, the electrochemical potential $\mu = \epsilon_F + eV_y = \epsilon_F + e\rho_{xy}J_x$, rather than the bottom of the energy subband, is assumed to remain constant over y.

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