Electron temperature in a quantum well. Energy loss caused by optical phonons

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The critical electron densities N_C^- and N_C^+ are determined. Above these electron densities, the electron temperature in the quantum well stabilizes below and above the threshold for the emission of an optical phonon, respectively. For the density range $N_C^- \ll N \lesssim N_C^+$, the distribution function is calculated near the threshold, where it differs markedly from the Maxwellian function.

The electron temperature T_e, which is higher than the lattice temperature T, is often used to describe a nonequilibrium electron gas in semiconductors. The superheating $T_e - T$ can be determined from the balance $P = Q_L$, where P is the energy which the electron gas receives from the external field or through optical pumping, and Q_L is the energy transferred to phonons. If the scattering by optical phonons $\hbar\Omega_0$ is appreciable, Q_L cannot be calculated in a nontrivial manner. If there is no interaction with phonons $\hbar\Omega_0$, the Fermi distribution $f_{T_e}(\epsilon)$ is established at relatively small electron densities, as soon as $\tau_{ee} \ll \tilde{\tau}_A$, where τ_{ee} is the scale time of electron-electron scattering, and $\tilde{\tau}_A$ is the scale time for the energy relaxation of electrons with acoustic phonons. The emission of phonons $\hbar\Omega_0$ depletes the $f_{T_e}(\epsilon)$ distribution above the threshold $\epsilon = \hbar\Omega_0$, and the electron-electron scattering depletes the distribution just below it as it strives to restore the distribution $f_{T_e}(\epsilon)$ through the flow across the threshold. If $T_e \ll \hbar\Omega_0$, then the distortion of the distribution is "integrally" weak, since it affects a region in which there are only a few electrons. In this region, the actual $f(\epsilon)$ may, however, differ markedly from $f_{T_e}(\epsilon)$. On the other hand, the region $\epsilon > \hbar\Omega_0$ determines Q_L.

The form that $f(\epsilon)$ assumes near the threshold is regulated by the kinetic equation

$$C_{ee}(f, f\,|\,\epsilon) - \Theta(\epsilon - \hbar\Omega_0)f(\epsilon)/\tau_0(\epsilon) = 0. \tag{1}$$

Here C_{ee} is the electron-electron collision integral, Θ is a step function, and τ_0 is the scale time for the emission of a phonon $\hbar\Omega_0$. The probability for the electron-electron scattering $W_{ee}(f\,|\,\epsilon \to \epsilon')$, which depends on f in a functional manner, is taken into account in the collision integral C_{ee}. Since the distortion of $f(\epsilon)$ is integrally small, we can calculate W_{ee} by setting $f = f_{T_e}$. A problem of this sort was solved for a 3D gas in Refs. 1–3. Since T_e was measured in quantum wells,[4–7] we considered it important to find $f(\epsilon)$ and Q_L for a nondegenerate 2D gas. In the latter case, the problem is much more complex, because, as we will show below, C_{ee} in a 2D gas is inconsistent with the Fokker–Planck differential representation, similar to that derived by Landau[8] for a 3D gas.

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We assume below that all electrons are situated at the lower level E_1 of the well and that they have an energy $\epsilon = \hbar^2 k^2/2m$ which is lower than the distance to the next level E_2. We would then have

$$W_{ee}(\epsilon \to \epsilon') = \frac{4\pi^{3/2}\hbar^3 E_B N}{L^2 (mT_e)^2}$$

$$\times |\omega|^{-3/2} e^{\omega/2} \int_{\gamma}^{\gamma^{-1}} \frac{du}{u^2} [(u^2 - \gamma^2)(\frac{1}{\gamma^2} - u^2)]^{-1/2} \exp\left\{-\frac{|\omega|}{4}(u^2 + \frac{1}{u^2})\right\}. \qquad (2)$$

Here L is the normalization length, E_B is the Bohr energy, $\omega = (\epsilon - \epsilon')/T_e$, $\gamma = |(k - k')/(k + k')|^{1/2}$, and $N - 2D$ is the electron density.

Using (2) to calculate the loss power level $Q_{ee}(\epsilon)$ at $T_e = 0$ and representing it as $\epsilon/\tau_{ee}(\epsilon)$, we find

$$1/\tau_{ee}(\epsilon) = \pi^2 \hbar E_B N / m\epsilon\,. \qquad (3)$$

A direct calculation yields

$$\frac{1}{\overline{\tau}_A(\epsilon)} = \frac{1}{\overline{\tau}_A} \frac{2ms^2}{\epsilon}$$

$$\frac{1}{\overline{\tau}_A} = \frac{\alpha}{(p_0 d)^3} \frac{1}{\overline{\tau}_{DA}} + \frac{\beta}{p_0 d} \frac{1}{\overline{\tau}_{PA}}\,. \qquad (4)$$

Here s is the velocity of sound $\hbar^2 p_0^2 = 2m\hbar\Omega_0$, $\overline{\tau}_{DA}$ and $\overline{\tau}_{PA}$ are the nominal times of deformation and polarization scattering,[9] and α and β are form factors (for a square well with infinitly high walls $\alpha = \pi^3/2$ and $\beta = 3\pi/4$). Comparing (3) and (4), we see that the Fermi distribution $f_{T_e}(\epsilon)$ establishes itself below the threshold at some distance from it if $N \gg N_C^-$, where

$$N_C^- = \frac{p_0^2}{2\pi^2} \frac{E_B/\hbar}{\overline{\tau}_A} \frac{2ms^2}{\hbar\Omega_0}\,. \qquad (5)$$

In (1) the region $|\epsilon - \hbar\Omega_0| \ll \hbar\Omega_0$ is large, where $\tau_0(\epsilon)$ is independent of ϵ, and W_{ee} in (2) becomes

$$W_{ee}(\epsilon \to \epsilon') = \frac{1}{\overline{\tau}_{ee}} \frac{4\pi^{-1/2}}{L^2 p_0^2} \left(\frac{T_e}{\hbar\Omega_0}\right)^{-3/2} e^{\omega/2} |\omega|^{-1} K_1\left(\frac{1}{2}|\omega|\right), \qquad (6)$$

where K_1 is Macdonald's function, and $\overline{\tau}_{ee} = \tau_{ee}(\epsilon = \hbar\Omega_0)$. We see from (6) that $W_{ee} \sim |\omega|^{-2}$ in the limit $\omega \to 0$. For such a Coulomb singularity all the probability moments of W_{ee} converge and the expansion in the moments cannot be cut off. (We recall that in 3D the first two moments diverge, and after the cutoff, the singularities are proportional to the large Coulomb logarithm, whereas the higher-order moments converge.) Equation (1) is therefore essentially an integral equation which can be solved by the Wiener–Hopf method.

After transforming to the variable $t = (\epsilon - \hbar\Omega_0)/T_e$, Eq. (1) will contain a single dimensionless parameter

$$\lambda = \pi^{-1/2} (\tau_0 / \bar{\tau}_{ee})(T_e / \hbar\Omega_0)^{-1/2} \equiv N / N_C^+ \tag{7}$$

At $\lambda \gg 1$ we find $f(\epsilon) \approx f_{T_e}(\epsilon)$. We thus see that

$$N_C^+ = \frac{p_0^2}{2\pi^{3/2}} \frac{E_B/\hbar}{\tau_0} \left(\frac{T_e}{\hbar\Omega_0}\right)^{1/2} \tag{8}$$

is that critical density above which the distribution is Maxwellian for all values of ϵ. If, on the other hand, $\lambda \ll 1$, then we would have

$$\frac{f(\epsilon)}{f_{T_e}(\epsilon = \hbar\Omega_0)} = \begin{cases} (2\lambda)^{1/2}, & |t| \ll \lambda \\ 2\pi^{-1/2}\lambda t^{-1/2} e^{-t}, & t > 0,\ t \gg \lambda \\ e^{-t}\, erf|t|^{1/2}, & t < 0,\ |t| \gg \lambda \end{cases} . \tag{9}$$

We see that below the threshold the distribution function $f(\epsilon)$ is distorted near the width of T_e. Above the threshold, the function $f(\epsilon)$ differs markedly from $f_{T_e}(\epsilon)$ in all cases, and there are fewer electrons by a factor of λ. The penetration depth of electrons into the region $\epsilon > \hbar\Omega_0$ is on the order of T_e, which is greater than the corresponding depth[2,10] for 3D. This holds despite the fact that in the 2D case the threshold for the phonon emission $\hbar\Omega_0$ is more rigid than that in the 3D case (the square root is replaced by a step), since in the 2D case the electrons enter the region $\epsilon > \hbar\Omega_0$, not through diffusion across the threshold but instead are driven into it from the region $\epsilon < \hbar\Omega_0$, accompanied by a "large" $\epsilon' - \epsilon \simeq T_e$ transfer. The loss power with optical phonons per electron is

$$Q_L = \frac{\hbar\Omega_0}{\tau_0} e^{-\hbar\Omega_0/T_e} \Phi(\lambda). \tag{10}$$

The general expression for $\Phi(\lambda)$ is quite cumbersome. Its asymptotic behavior is

$$\Phi(\lambda) = \begin{cases} 2\lambda, & \lambda \ll 1 \\ 1 - (\pi\lambda)^{-1}, & \lambda \gg 1 . \end{cases} \tag{11}$$

Finally, we find some estimates for a well of width $d = 150$ Å in GaAs, with $E_2 - E_1 = 2\hbar\Omega_0$. Using[11] $\tau_0 = (2/\pi)\bar{\tau}_{PO}$ is the nominal time for scattering by *LO* phonons,[9] at $T_e = 20$ K from (8) we find $N_C^+ \approx 2\times10^{11}$ cm^{-2} and from (5) we find $N_C^- \approx 3\times10^6$ cm^{-2}. The scale time for the establishment of the electron temperature $T_e = 20$ K, with $N = 10^{10}$ cm^{-2}, is 0.2 ps.

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