Beta decay under the influence of a quantizing magnetic field

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Quantum effects which accompany the beta decay and which arise in a strong magnetic field under conditions that produce a highly dense relativistic electron gas are studied.

The study of many features of the production of neutron stars requires the use of several extremal parameters which determine the conditions under which the physical processes occur at the collapse stage.[1]

Theoretical and experimental studies of neutrino reactions show that they may play a key role in the evolution of stars and in supernova explosions. These reactions are responsible for most of the irreversible energy loss of stars at high densities and high temperatures. Isolated studies of the so-called URKA processes

$$n \to p + e^- + \tilde{\nu}, \qquad (1)$$

$$p + e^- \to n + \nu \qquad (2)$$

in a strong magnetic field have been reported previously.[2,3] In this letter we report the results of an ongoing study of essentially quantum effects in reactions (1) and (2) which occur in intense magnetic fields at high density of the relativistic electron gas.

Quantization of the transverse momentum of light charged particles in a magnetic field H shows that in the case of a strong degeneracy ($kT \ll \mu mc^2$, where μ is the chemical potential) the basic parameters are the integral parts of the quantities $N_1 = H_c(\mu^2 - 1)/(2H)$, $N_2 = H_c(\epsilon_0^2 - 1)/(2H)$, where $H_c = m^2c^3/(e\hbar)$. These quantities essentially determine the number of allowed Landau levels present during the energy evolution $\epsilon_0 = (M_n - M_p)/m$. As in the other processes occurring in a magnetic field, with $N_1, N_2 \gg 1$ (many levels are populated), the quantizing nature of the field is hardly evident, whereas at $N_1, N_2 \sim 1$ the quantum effects become more pronounced.

In the absence of a magnetic field, the probability for the occurrence of processes (1) and (2) in the limit $\mu \to \epsilon_0$ decreases monotonically (see Fig. 1), principally because of the reduction of the spectral interval of the electrons that participate in the reactions.

In the case of a completely degenerate electron gas, the probability for the occurrence of processes (1) and (2) in a strong magnetic field can be written as follows:

$$W^{(1)}(H)/W_0 = H(F_1 - F_2)/H_c, \quad (\mu < \epsilon_0), \qquad (3)$$

$$W^{(2)}(H)/W_0 = H(F_2 - F_1)/H_c, \quad (\mu > \epsilon_0), \qquad (4)$$

where

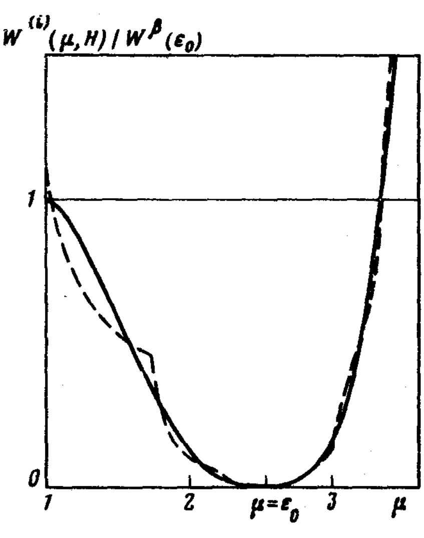

FIG. 1. A plot of β-decay probabilities normalized to the β-decay probability in a vacuum (1) and of the capture (2) in a relativistic degenerate electron gas in a magnetic field $H=0$(—) and $H/H_c = 1$(- - - -) versus the chemical potential μ.

$$F_1(\epsilon_0) = \sum_{n=0}^{[N_2]} (1 - \delta_{n,0}/2) I(\epsilon_0, b_n),$$

$$F_2(\mu) = \sum_{n=0}^{[N_1]} (1 - \delta_{n,0}/2) I(\mu, b_n),$$

$$I(q, b_n) = b_n^3\{s^3/3 + s[1 - \epsilon_0(q - \epsilon_0)/b_n^2] - (\epsilon_0/b_n)\ln(q/b_n + s)\},$$

$$s^2 = (q/b_n)^2 - 1,\ b_n^2 = 1 + 2nH/H_c,\ W_0 = G^2 m^5 (1 + 3\alpha_0^2)/(2\pi^3),$$

$\alpha_0 = G_A/G_V$, G is the Fermi constant, and $\delta_{n,0}$ is the Kronecker symbol. It is easy to see from expressions (3) and (4) that in a sufficiently strong magnetic field $H > H_c(\epsilon_0^2 - 1)/2$, $H_c(\mu^2 - 1)/2$ only single terms with the index $n = 0$ remain in the sums that determine the functions F_i (the electrons occupy the ground state). The total probability for the occurrence processes (1) and (2) increases linearly with increasing magnetic field strength, since the field dependence drops out in the functions F_i:

$$(F_1(\epsilon_0) - F_2(\mu))|_{H=0} = \frac{1}{2}\{[(\epsilon_0^2 - 1)^{3/2} - (\mu^2 - 1)^{3/2}]/3 + (\epsilon_0^2 - 1)^{1/2}$$

$$- (\mu^2 - 1)^{1/2}(1 - \epsilon_0\mu + \epsilon_0^2) - \epsilon_0 \ln[(\epsilon_0 + (\epsilon_0^2 - 1)^{1/2})/(\mu + (\mu^2 - 1)^{1/2})]\}.$$

The monotonic decrease of the probability remains in force in this limit as $\mu \to \epsilon_0$.

In general, an analysis of the expressions for the total probability for the occurrence of processes (1) and (2) as a function of H/H_c shows that F_i are rather complex field functions which contain, along with monotonically varying components, some oscillating components that arise due to the threshold peculiarities of the magnetic field. Restricting the discussion here to the terms of order $(H/H_c)^4$, we can represent the functions F_i as follows:

$$\begin{aligned} F_i(q)H/H_c &= \Phi_0(q) + (H/H_c)^2 \Phi_1(q)/3 - (H/H_c)^4 \Phi_2(q)/360 \\ &+ (q - \epsilon_0)^2 (2H/H_c)^{3/2} \zeta(-1/2, v) + [2(\epsilon_0 - q)/(3q)](2H/H_c)^{5/2} \zeta(-3/2, v) \\ &+ [2\epsilon_0/(15 q^3)](2H/H_c)^{7/2} \zeta(-5/2, v), \end{aligned} \quad (5)$$

where $\zeta(s,v)$ is a generalized zeta function, v is the fractional part of N_i, and

$$\Phi_0(q) = (\epsilon_0/2)\ln[q + (q^2 - 1)^{1/2}] + (q^2 - 1)^{1/2}(-4/15 - 2\epsilon_0^2/3$$

$$+ \epsilon_0 q/2 - 2q^2/15 + 2\epsilon_0^2 q^2/3 - \epsilon_0 q^3 + 2q^4/5),$$

$$\Phi_1(q) = \epsilon_0 \ln[q + (q^2 - 1)^{1/2}] + (q^2 - 1)^{-1/2}(1 - q^2/2 - \epsilon_0 q + \epsilon_0/2),$$

$$\Phi_2(q) = (q^2 - 1)^{-5/2}(4\epsilon_0 q^5 - 10\epsilon_0 q^3 + 5q^2 + 3\epsilon_0^2 - 2).$$

The expansion

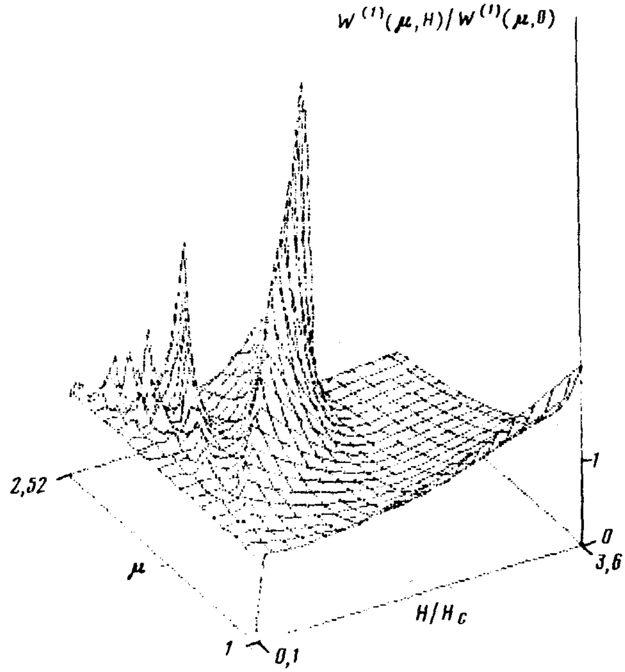

FIG. 2. A plot of the β-decay probability (1), scaled to the probability of the same process at $H=0$, versus the magnetic field H/H_c and the chemical potential μ.

$$\zeta(s,v) = 2(2\pi)^{s-1}\Gamma(1-s)\sum_{n=1}^{\infty} n^{s-1}\sin[2\pi n v + \pi s/2], \qquad (6)$$

is valid[4] for the function $\zeta(s,v)$. It is clear from this expansion that the components proportional to this function are of an oscillating nature. Qualitatively, the particular features of the oscillations are determined by the first term of expansion (6); however, the accuracy can be improved through calculations on a computer. This expression

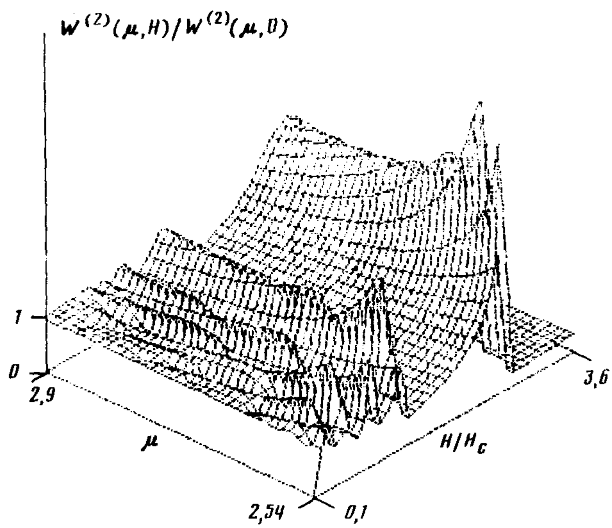

FIG. 3. A plot of the probability for the capture reaction (2), scaled to the probability for the same process at $H=0$, versus the magnetic field H/H_c and the chemical potential μ.

works well for $N_1, N_2 \gg 1$. If, on the other hand, N_1 and N_2 are small, expressions (3) and (4) may prove to be more useful in calculating the total probability.

A similar behavior of the probability for the β decay in the external field of an electromagnetic wave was pointed out elsewhere,[5] but the oscillating corrections in a wave field turned out to be largely suppressed monotonic contributions. In the case studied by us, the role of the field corrections increases markedly, and for certain values of the magnetic field H in the limit $\mu \to \epsilon_0$ the oscillating terms become dominant even in comparison with the vacuum contribution (see Figs. 2 and 3).

For situations that may arise during a collapse, these studies show that the neutrino yield from regions of a star of various densities and various magnetic fields is not uniform. This effect may give rise, for example, not only to a recoil momentum of pulsars along the magnetic-field direction[3–6] but also to an additional change in their direction.[7,8]

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