Interaction of a magnetic monopole with a material medium

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The basic relations for the electrodynamics of a magnetic monopole in a medium must be reexamined. The energy loss of a magnetic monopole in classical media is found. It is noted in particular that there is no longitudinal energy loss. A special "explosive" mechanism for the energy loss of magnetic monopoles is described.

The electrodynamics of the magnetic monopole underlies methods for detecting magnetic monopoles, for estimating upper limits on their flux density, etc. In this letter we discuss a macroscopic formulation of this electrodynamics, which in principle offers the most general and unified description of the interaction of a magnetic pole with a medium (the medium is assumed to be an isotropic, nongyrotropic, homogeneous, equilibrium medium which contains no magnetic monopoles).

1. This formulation of electrodynamics is usually based on Maxwell's equations,

$$\begin{aligned} &\operatorname{curl} \mathbf{B} - \dot{\mathbf{E}} = \mathbf{j} + \mathbf{j}' \quad \text{a}, && \operatorname{div} \mathbf{E} = \rho + \rho' \quad \text{b}, \\ &-\operatorname{curl} \mathbf{E} - \dot{\mathbf{B}} = \tilde{\mathbf{j}} \quad \text{c}, && \operatorname{div} \mathbf{B} = \tilde{\rho} \quad \text{d}; \end{aligned} \tag{1}$$

the standard expressions for the induced charge and current densities,

$$\rho' = (1-\epsilon)\operatorname{div}\mathbf{E}, \qquad \mathbf{j}' = (\epsilon - 1)\dot{\mathbf{E}} + (1 - {}^{1}/_{\mu})\operatorname{curl}\mathbf{B}; \tag{2}$$

and an expression for the energy loss,

$$W = -\dot{E}_{kin} = -\int dx\,(\mathbf{j}\mathbf{E} + \tilde{\mathbf{j}}\mathbf{H}), \qquad \mathbf{H} = \mathbf{B}/\mu, \tag{3}$$

where ρ and $\mathbf{j}$ are the external charge density and the external current density in Heaviside units, $\tilde{\rho}$ and $\tilde{\mathbf{j}}$ are the same quantities for the magnetic monopole, ϵ and μ are the dielectric constant and permeability, and $c = 1$.

The fields $\mathbf{E}$ and $\mathbf{B}$ are determined by their action on a classical charge:

$$m\dot{\mathbf{v}} = e(\mathbf{E} + [\mathbf{v}\mathbf{B}]), \tag{4}$$

having the Fourier components $\mathbf{E}_l = -iD_l\mathbf{k}\rho$, $\mathbf{B}_l = -i\mathbf{k}\tilde{\rho}/k^2$,

$$\mathbf{E}_t = iD_t(\omega\mathbf{j}_t - [\mathbf{k}\tilde{\mathbf{j}}]/\mu), \quad \mathbf{B}_t = iD_t([\mathbf{k}\mathbf{j}] + \omega\epsilon\tilde{\mathbf{j}}_t), \tag{5}$$

where l and t specify the components longitudinal and transverse with respect to k, and

$$D_l = (k^2\epsilon)^{-1}, \qquad D_t = (k^2/\mu - \omega^2\epsilon)^{-1}$$

are the components of the photon Green's function in the medium.

2. From (3) and (5) we find a specific "explosive" mechanism for the energy loss of a magnetic monopole: a rapid conversion of its kinetic energy E_{kin} into field energy (radiation or heat). This is true of a nonideal plasma, strong electrolytes, several simple metals, and other media for which ϵ and $1/\mu$ have a pole at $\omega = i\Omega$ (although the functions D_l, D_t, and $1/D_t$ are analytic in the upper ω half-plane).[1] A causal circumvention of this pole[2] leads to increases in the field $\mathbf{E}_t$ and $\mathbf{B}_t$ and also in the loss W in an $\exp(\Omega t)$ manner.

The obvious importance of this effect, which follows directly from (2) and (3), turns our attention to a critical analysis of these relations.

3. Some doubt arises in this connection even when we apply (2) to a superconductor, were we have $\omega\epsilon \to 0$, $\mu \to k^2\lambda^2$, $D_t \to \lambda^2$ in the limit $k \to 0$ (far from a field source) and $\omega/k \to 0$, where λ is the London penetration depth. It would follow from (5) that there is no Meissner effect for the magnetic-monopole fields $\mathbf{E}_t$ and $\mathbf{B}$, i.e., that superconducting detectors of magnetic monopoles would be totally ineffective.[1)]

4. It can indeed be shown that Eqs. (2) and (3) are not appropriate. In the absence of a magnetic monopole, expression (2) for $\mathbf{j}'$ is merely one of a set of equivalent expressions differing in the form of the transverse component:

$$\mathbf{j}_t' = (\tilde{\epsilon} - 1)\dot{\mathbf{E}}_t + (1 - 1/\tilde{\mu})\,\mathrm{curl}\,\mathbf{B}, \tag{2a}$$

where $\tilde{\epsilon}$ and $\tilde{\mu}$ are arbitrary but related by $k^2/\tilde{\mu} - \omega^2\tilde{\epsilon} = D_t^{-1}$. In addition to the choice $\tilde{\epsilon} = \epsilon, \tilde{\mu} = \mu$, which corresponds to (2), the following expressions are frequently used[4]:

$$\tilde{\epsilon} = \epsilon_t = \epsilon + (1 - 1/\mu)k^2/\omega^2, \quad \tilde{\mu} = 1. \tag{6}$$

This uncertainty is a consequence of the rigid coupling in (1c) of the fields **E** and **B**, which allows a regrouping of the **j**′ terms in (2).

A magnetic monopole disrupts this coupling, fixing $\tilde{\epsilon}$ and $\tilde{\mu}$, which become additional characteristics of the medium (along with ϵ and μ). This conclusion can be seen directly from expressions (5) (with ϵ,μ replaced by $\tilde{\epsilon}$, $\tilde{\mu}$): The fields of the charge depend on only D_t, while the fields of the magnetic monopole also depend on $\tilde{\epsilon}$ and $\tilde{\mu}$.

5. That (3) is inappropriate can be seen even in the case of classical media, to which we restrict the rest of the paper. The energy loss in an extended medium which absorbs all the radiation from the source is equal to the amount of work performed on the medium, $\int d\mathbf{x}\,\mathbf{j}'\mathbf{E}$. Using (1) and omitting the quantity $\int d\mathbf{x}(E^2 + B^2)$, we replace (3) with

$$W = -\int d\mathbf{x}\,(\mathbf{j}\mathbf{E} + \tilde{\mathbf{j}}\mathbf{B}). \tag{3a}$$

The standard derivation of (3) uses the expression $\int d\mathbf{x}(\mathbf{E}\delta\mathbf{D} + \mathbf{H}\delta\mathbf{B})$ for the variation of the field energy in the medium; this expression is correct only in the absence of magnetic monopoles.[2)]

It follows from (3a) [in contrast with (3); see Ref. 5] that a magnetic monopole suffers no longitudinal energy loss: The corresponding term of W does not depend on the properties of the medium.

6. In the case of a classical medium, $\tilde{\epsilon}$ and $\tilde{\mu}$ agree with (6); i.e., for such a medium, according to (2a), we have $\mathbf{j}' = 0$ as the field **B** acts. Actually, this field does not contribute to the Liouville equation $\dot{f} + \Sigma(\mathbf{v}\nabla + \dot{\mathbf{v}}\nabla_{\mathbf{v}})f = 0$, according to (4), since we have $\nabla_{\mathbf{v}} f \propto \mathbf{v}$ for an equilibrium distribution function f (the summation is over the particles of the medium).[3)]

For a superconductor, thought of as a classical ideal liquid, we have $\tilde{\epsilon} = \epsilon_t \to (\omega\lambda)^{-2}$ and $\tilde{\mu} = 1$, which lead to a Meissner effect with all its consequences (Sec. 3).

7. Using (3a) and (6), we find the transverse energy loss of a magnetic monopole to be

$$W = \frac{2g^2c^2v}{\pi}\int_0^\infty dk k^3 \int_0^{kv} \frac{d\omega}{\omega}\left(1 - \frac{\omega^2}{k^2v^2}\right)\mathrm{Im}D_t, \tag{7}$$

where g and **v** are the charge and velocity of the monopole; here and below, we use standard units. For equal velocities of the magnetic monopole and the charge ($\overline{W}$ is the transverse energy loss of the charge) we have

$$W \geqslant (g^2c^2/e^2v^2)\overline{W}.$$

The Čerenkov energy loss of a magnetic monopole (in the absence of spatial dispersion),

$$W = \frac{g^2}{c}\int_0^\infty d\omega\,\omega[\tilde{\epsilon}(\omega) - c^2/(v^2\mu(\omega))]\,\mu^2(\omega),$$

differs by a factor of μ^2 from the standard expression based on (2) and (3). In the case of low velocities, on the other hand, expression (7) gives us

$$W = \frac{16g^2v^2}{3c^2}\int_0^\infty dk\,\sigma_t(0,k),$$

where $\sigma_t(\omega,\mathbf{k}) = \omega \mathrm{Im}\epsilon_t/4\pi$ is the transverse conductivity of the medium. A similar response[5] (without stipulations that the medium is classical) was found without the use of expressions (2) and (3), which would lead to a dependence $W \propto v^4$.

8. In classical media, the quantity $\tilde{\epsilon} = \epsilon_t$ is analytic in the upper ω half-plane along with $1/D_t$, so that there can be no "explosion" of a magnetic monopole (Sec. 2). In the quantum-mechanical case, in contrast—the case of particular interest to us—this effect remains an open question, since (3a) cannot be used. An affirmative answer would lead to an ideal detector of magnetic monopoles.

9. We note in conclusion that only in media with $\tilde{\epsilon} = \epsilon_t$, $\tilde{\mu} = 1$ could a magnetic monopole, i.e., a source of a longitudinal magnetic field, be simulated by a thin solenoid (carrying a current independent of the properties of the medium). The substitution $\mathbf{B} \rightarrow \mathbf{B} - \mathbf{B}_0$, where $\mathbf{B}_0 = \int dt\tilde{\mathbf{j}}$ is a string along the path of the magnetic monopole, eliminates the right sides of (1c) and (1d), adding to (1a) the current of the equivalent solenoid, $\mathrm{curl}\mathbf{B}_0/\tilde{\mu}$ [see (2a)].[6]

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[1)] Their theory is based on dynamic equations,[3] not on Eqs. (1) and (2).
[2)] Expression (3a), which can be derived by a microscopic approach, incorporates statistical fluctuations.
[3)] The fact that (4) is not of a Hamiltonian nature in the presence of magnetic monopoles is not an obstacle to the derivation of the Liouville equation.

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