Structure functions of deep inelastic scattering at intermediate *x* in quantum chromodynamics

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The contribution of valence quarks to the structure function of deep inelastic νp scattering at intermediate x and $Q^2 = 5$–$20\ \mathrm{GeV}^2$, i.e., the distribution of valence d quarks in the proton, $d_V(x)$, is calculated in quantum chromodynamics. The calculation results which lack adjustable parameters of any sort are in agreement with the experiment.

One of the most interesting and important problems posed by quantum chromodynamics is the problem of finding the structure functions of deep inelastic scattering of leptons by hadrons, which is equivalent, according to the quark-parton model, to finding the quark distributions in hadrons. The quest for the solution of this problem so far has had only limited success: The second-moments of the structure functions of the nucleon and pion[1,2] and their behavior[3] at small x were determined by the operator-expansion method (the QCD sum rules) (a summary of the results obtained by using this method was published by Ioffe[4]). The application of this method to calculating the structure functions runs into serious difficulties, since this method is not suitable either at small x or large x. (As usual, $x = Q^2/2\nu$, Q^2 is the square of the momentum transferred to the hadron, $Q^2 = -q^2$, ν/m is the energy transfer, and m is the hadron mass.)

Kolesnichenko[1] and Belyaev and Blok,[2] who studied the second moments of the structure functions, were able to overcome these difficulties by introducing new vacuum condensates into the analysis. The use of this approach in the case of higher moments is, however, of highly dubious value.

In the present letter we propose a method of finding the structure functions at intermediate $x (0.1 < x < 0.5)$. The capability of this method can be demonstrated through a calculation of the contribution of valence quarks to the function $F_2^{\nu p}(x)$ of the deep inelastic νp scattering which is related to the distribution of valence d quarks in the proton by (see Ref. 9) $F_{2V}^{\nu p}(x) = 2x d_V(x)$.

Let us consider the four-point amplitude

$$T_{\mu\nu}(p,q) = -i\int d^4x\, d^4y\, d^4z\, e^{iqx}\, e^{ip(y-z)} < 0 \,|\, T\{ j_\mu^-(x), j_\nu^+(0), \eta(y), \bar{\eta}(z)\} \,|\, 0 >, \tag{1}$$

where $j_\mu = \bar{d}\gamma_\mu(1+\gamma_5)u$ and $j_\mu^+ = (j_\mu^-)^+$ are the weak quark currents, and

$$\eta = u^a C \gamma_\lambda u^b \gamma_5 \gamma_\lambda d^c \epsilon^{abc} \tag{2}$$

is the three-quark current with the proton quantum numbers.[6] We assume that

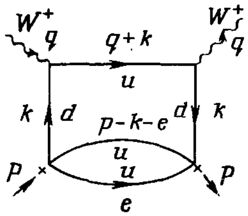

FIG. 1.

$p^2<0, |p^2| \sim 1\,\mathrm{GeV}^2$, $q^2<0$, and $|q^2| \gg |p^2|$. We will calculate $\mathrm{Im}T_{\mu\nu}$ (p,q) by means of the operator expansion, retaining only the terms of the first nonvanishing approximation in the expansion in powers of p^2/q^2. We restrict the analysis to the region $5 \lesssim Q^2 \lesssim 20$ GeV^2, in which the perturbative corrections reduce to a renormalization of the structure functions. A highly simplified diagram corresponding to $T_{\mu\nu}$ in (1) is shown in Fig. 1.

The main problem in calculating $T_{\mu\nu}$ by the operator-expansion method is that the total momentum in the t channel is zero, which may give rise to singularities in the limit $t \to 0$. This problem can, however, be completely eliminated in the analysis of $\mathrm{Im}T_{\mu\nu}$. Studying $\mathrm{Im}T_{\mu\nu}$ for the diagram in Fig. 1 (and also for more complex diagrams), we see that at $p^2<0$ $\mathrm{Im}T_{\mu\nu}$ is analytic with respect to t at small values of t, and the virtual nature of the quark which interacts with the W boson is strong at intermediate x: $k^2 \sim p^2 x(1-x)$.

We restrict the analysis to the structures in $T_{\mu\nu}$ which conserve chirality. For this purpose, we will calculate Sp $(\hat{p}\mathrm{Im}T_{\mu\nu})$. In constructing the operator expansion, in addition to the principal term corresponding to the diagram in Fig. 1, we will use the power-law correction due to the quark condensate, which is proportional to $\alpha_s\langle 0|\bar{\psi}\psi|0\rangle^2$ (the four-quark vacuum expectation values are assumed to be factorized). The correction $\sim\alpha_s\langle 0|\bar{\psi}\psi|0\rangle^2$ is very important, since it defines the range of permissible values of p^2 and x. The result of the calculations is

$$\mathrm{Sp}\,(\hat{p}\,\mathrm{Im}\,T_{\mu\nu}) = \frac{2}{\nu}\Bigg\{-(2\pi)^{-3}p^4\ln(-p^2)(1-x)(2+2x-x^2)$$
$$+\frac{8}{9}\alpha_s<0|\bar{\psi}\psi|0>^2\frac{1}{p^2}\left[\left(2x\ln\frac{2\nu}{-p^2x}-\frac{7}{4}+\frac{x}{2}\frac{1}{1-x}\right)+1+3x\right.$$
$$\left.+(1-x)\ln\frac{2\nu}{-p^2x}\right]\Bigg\}\left(2xp_\mu p_\nu+p_\mu q_\nu+p_\nu q_\mu-\nu\delta_{\mu\nu}\right)+\frac{16}{9}\alpha_s<0|\bar{\psi}\psi|0)^2$$
$$\times\frac{1}{p^2\nu}(q_\mu q_\nu-q^2\delta_{\mu\nu}). \qquad (3)$$

In (3) we have retained only the terms which are singular in p^2 as $p^2\to 0$. In calculat-

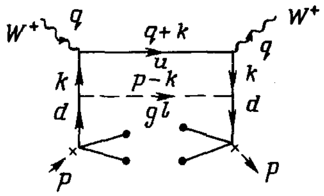

FIG. 2.

ing the power-law correction we used only the diagrams with rigid gluons, such as that in Fig. 2. The diagrams with soft gluons turned out to be numerically small. As we can see in Fig. 3, at large x the ratio of the power-law correction to the principal term increases as $(1-x)^{-2}$, limiting the range of applicability of this approach at large x.

$\mathrm{Im}T_{\mu\nu}$ can be expressed in terms of the physical states. Writing the dispersion relation in terms of p^2 for the invariant function of the structure $\sim p_\mu p_\nu$ and singling out in it the proton component, we find

$$\mathrm{Sp}\,(\hat{p}\,\mathrm{Im}\,T_{\mu\nu}) = -\,8\pi^2\lambda_N^2\, w_2^{\nu p}(\nu, x)\left[\frac{1}{(p^2-m^2)^2} + \frac{A}{p^2-m^2} + B\right] \times \left(p_\mu - \frac{\nu q_\mu}{q^2}\right)\left(p_\nu - \frac{\nu q_\nu}{q^2}\right) + \text{subtr. terms.} \tag{4}$$

(The terms in the structure $q_\mu q_\nu - q^2\delta_{\mu\nu}$ were dropped.) The term with a first-order pole in p^2-m^2 corresponds to nondiagonal transitions $Wp \to WN^*$ (the diagram in Fig. 3) and B corresponds to the continuum. The amplitude of the proton transition to the quark current, λ_N, was determined in Refs. 6 and 7: $\tilde{\lambda}_N^2 = 32\pi^4\lambda_N^2 = 2.1$ GeV (Ref. 6).

We equate the structure functions in (3) and (4), use in this relation the Borel transformation with the parameter M^2, and differentiate the result with respect to $1/M^2$ in order to eliminate the unknown constant A. We thus find

$$F_2^{\nu p}(x) = e^{m^2/M^2}(\tilde{\lambda}_N^2 m^2)^{-1} x M^8 [3E_3(z) - (m^2/M^2)E_2(z)]\,4(1-x)(2+2x-x^2) + \frac{8}{9\pi} m^2 \alpha_s a^2 \left[-\left(\frac{7}{4} - \frac{x}{2}\right)\frac{1}{1-x} + 1 + 3x + \left(\frac{2x}{1-x} + 1 - x\right)\left(\ln\frac{2\nu}{M^2 x} + \frac{M^2}{m^2} + C\right)\right\}, \tag{5}$$

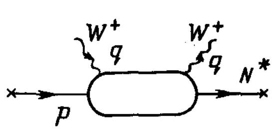

FIG. 3.

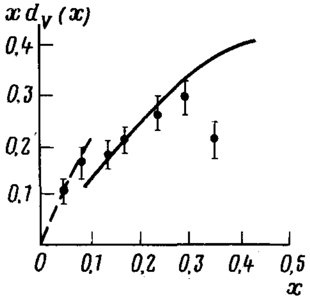

FIG. 4.

where m is the nucleon mass, and $C = 0.577$ is Euler's constant,

$$E_2(z) = 1 - \left(1 + z + \frac{z^2}{2}\right)e^{-z}, \quad E_3(z) = 1 - \left(1 + z + \frac{z^2}{2} + \frac{z^3}{6}\right)e^{-z} .$$

Here $z = W^2/M^2$, W is the continuum threshold, $W^2 \approx 2.3$ GeV^2 (Ref. 7), $a = (2\pi)^2 \langle 0|\bar{\psi}\psi|0\rangle^2$, and $\alpha_s a^2 = 0.13$ GeV (Refs. 4 and 6). Examining the M^2 dependence on the right side of (5), we see that the most suitable values of M^2 are, as in the case of the sum rules for the mass[6] and the magnetic moments of nucleons,[7] in the region $M^2 \approx 1$ GeV^2. Here Eq. (5) is applicable only at $x < 0.4$–0.5, since at $x = 0.5$ the power-law correction can be as high as 30%.

Expression (5) corresponds to the valence-quark component in $F_2^{\gamma p}(x)$, since the quark-sea component (the diagram with loops) was ignored in calculating it. Figure 4 is a comparison of the distribution of valence d quarks in the proton, $xd_V(x)$, calculated according to (5) (solid curve), with the experimental data of Ref. 8. The dashed curve is the distribution obtained in Ref. 3 at small x. Since we ignored in our calculations other power-law corrections, in particular, the gluon condensate, the agreement with the experiment obtained by us may be considered satisfactory.

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