The contribution of surface fluctuations to the free energy of liquid ^{3}He

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We evaluate the contribution of surface fluctuations to the free energy of normal liquid ^{3}He and predict the temperature dependence of the coefficient of surface tension in the low-temperature region.

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The temperature dependence of the surface tension of ^{4}He at low temperatures is completely determined by the free energy of the capillary waves.[1] In normal liquid ^{3}He there are no surface disturbances of this type. At absolute zero ($T = 0$) the surface disturbances in a Fermi liquid are a purely diffusive mode due to the Landau damping mechanism. Their spectrum is purely a damped one: $\omega(\mathbf{k}) = -i\dfrac{4\delta_0}{3\rho_0 p_F}k^2$. Here we are considering a semi-infinite Fermi liquid for which $\mathbf{k}$ is a two-dimensional wave vector along the surface of the system, which lies on the zy plane, the x axis being perpendicular to the surface, and δ_0, ρ_0, and p_F are the coefficient of surface tension, the density, and the Fermi momentum, respectively. We shall evaluate the contribution of these surface fluctuations to the free energy at low temperatures by the method proposed by Larkin and Mel'nikov in Ref. 2.

The pole part of the scattering amplitude, which is due to the exchange of surface excitation, is of the form[3]

$$\Gamma_s(x, x', \mathbf{k}, \omega) = \frac{\Omega(x, \mathbf{k})\,\Omega(x', \mathbf{k})}{(\Omega_k^+\Omega_k) - (\Omega_k A(k, \omega)\Omega_k)}, \tag{1}$$

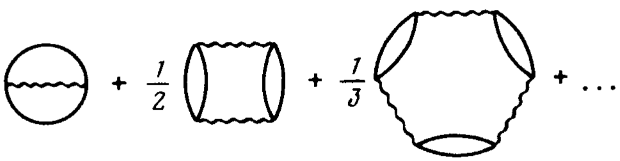
where $\Omega_k = \Omega(x,\mathbf{k})$, which is the amplitude for the creation of the excitation, satisfies the equation

$$\Omega_k = (F_k A(\mathbf{k}, \omega_k)\Omega_k), \tag{2}$$

$$A(x, x', \mathbf{k}, \omega) = \int d^2(r_\perp - r'_\perp)\exp\{-i\mathbf{k}(\mathbf{r}_\perp - \mathbf{r}'_\perp)\}\int\frac{\partial\epsilon}{2\pi i}G\left(\mathbf{r}, \mathbf{r}'; \epsilon + \frac{\omega}{2}\right)$$

$$\times G\left(\mathbf{r}', \mathbf{r}, \epsilon - \frac{\omega}{2}\right) \tag{3}$$

Here $\mathbf{k}$ is the harmonic of the particle-hole propagator, F_k is the $\mathbf{k}$-th harmonic of the effective interaction of the quasiparticles, which is given by a relation analogous to (3), Ω_k^+, which satisfies the adjoint of Eq. (2), is the transition frequency corresponding to amplitude Ω_k, and $\mathbf{r}_\perp = \{0,y,z\}$. The parentheses in Eqs. (1) and (2) and everywhere below denote integration over all intermediate x coordinates.



In order to evaluate the contribution of surface excitations to the free energy by the method of Ref. 2, one must sum the above series of ring diagrams, where the wavy lines are understood to mean the k-th limit of the amplitude: $\Gamma_s(x,x',k,0)$, and the particle-hole loops (GG) are taken as the frequency-dependent part of the propagator: $[A(\mathbf{k},\omega) - A(\mathbf{k},0)]$.

$$\delta F_s = \frac{1}{2}\Gamma\int\frac{d^2k}{(2\pi)^2}\,T\sum_n \ln\left\{1 - \frac{(\Omega_k[A(\mathbf{k},\omega_n) - A(\mathbf{k},0)]\Omega_k)}{(\Omega_k^+\Omega_k) - (\Omega_k A(\mathbf{k},0)\Omega_k)}\right\}, \tag{4}$$

where $\omega_n = 2\pi i n T$, and Γ is the surface area of the system. The temperature-independent part should be subtracted from Eq. (4). We shall therefore evaluate the contribution to the entropy $\delta S_s = -\partial(\delta F_s)/\partial T$ straightaway. The sum over n in Eq. (4) can be done in the usual way be replacing the sum by a contour integral over $d\omega$. The main contribution to the integral is from the frequency region $|\omega| \lesssim T \ll \epsilon_F$, in which

$$(\Omega_k^+ \Omega_k) - (\Omega_k A(\mathbf{k}, \omega)\,\Omega_k) \approx -\delta_0 k^2 + i\,\frac{3}{4}\rho_0 p_F\,\omega\,\mathrm{sgn}(\mathrm{Im}\,\omega). \quad (5)$$

Using this expression, one can write the entropy δS_s in the form

$$\delta S_s = \frac{\Gamma}{(4\pi)^2}\int_0^{k_D^2} dk^2 \int_0^\infty \frac{z}{\mathrm{sh}^2\frac{z}{2}}\,\mathrm{arctg}\,\frac{Tz}{|\omega(k)|}\,dz. \quad (6)$$

Because the integral over dk^2 diverges at the upper limit, one introduces a Debye-type cutoff at $k_D \sim p_F$ in Eq. (6). This cutoff is justified on physical grounds, since there are no deformations of the surface with periods shorter than the distance between particles $r_a \sim 1/p_F$. The correct specification of $(\Omega_k A(k,\omega)\Omega_k)$ for $k \sim p_F$ would permit calculation of k_D (and also the fluctuation spectrum $\omega(k)$ for large k). In our case, however, we are forced to consider k_D to be a phenomenological parameter.

After completing the integration in Eq. (6), we obtain an expression for the correction to the coefficient of surface tension $\delta\sigma_s$:$\delta\sigma_s = \Gamma d(\delta\sigma_s)/dT$ for low temperatures $T \ll \epsilon_F$:

$$\delta\sigma_s(T) = \frac{1}{32}\,\frac{\rho_0 p_F}{\sigma_0}\,T^2 \ln(T/\Theta_D), \quad (7)$$

where $\Theta_D = \frac{4}{3}\frac{\sigma_0}{\rho_0 p_F} k_D^2$. It is seen that the contribution of surface excitations (7) leads to a sharper dependence on T than does the "Fermi-gas" contribution, which goes as T^2, and is the dominant contribution at low temperatures. Thus, for $T \ll \epsilon_F$ the coefficient of surface tension of normal ^{3}He behaves as

$$\sigma(T) = \sigma_0\left[1 + \frac{1}{32}\,\frac{\rho_0 p_F}{\sigma_0^2}\,T^2 \ln(T/\Theta_D)\right]. \quad (8)$$

Such behavior of $\sigma(T)$ evidently should be manifested at $T < 0.1$ K, where liquid ^{3}He is well described by Landau theory. The existing experimental data[4] do not permit verification of the validity of this law.

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