Superfluidity of the vacuum near an anisotropic singularity: a new phase transition

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We examine a quantum scalar field with a $\lambda\phi^4$ interaction in an anisotropic space-time and show that for large enough anisotropy there is a phase transition leading to the formation of a condensate with energy density $\sim -1/\lambda t^4$.

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The quantum effects of particle production and vacuum polarization of a free scalar field of mass m in an anisotropic space-time of Bianchi type I with the metric

$$ds^2 = dt^2 - \sum_{\alpha=1}^{3} a_\alpha^2(t)(dx^\alpha)^2 \tag{1}$$

were examined in Refs. 1 and 2 (see also Ref. 3).

In this letter we show that for a field with self-action in this metric, if the anisotropy is sufficiently large there is a vacuum phase transition analogous to the transition of a Bose liquid to the superfluid state. The effect occurs for $Q > m^2$, where Q is

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the anisotropy parameter of metric (1):

$$Q = \frac{1}{18}[(h_1 - h_2)^2 + (h_2 - h_3)^2 + (h_3 - h_1)^2] \tag{2}$$

($h_\alpha = \dot{a}_\alpha / a_\alpha$ are the Hubble parameters).

A free quantized field in metric (1) can be described in terms of quasiparticles having Heisenberg creation and annihilation operators $c_{\mathbf{k}}^{(+)}(t)$ and $c_{\mathbf{k}}^{(-)}$. The field operator is

$$\phi(x) = \frac{1}{(2\pi v)^{3/2}} \int \frac{d^3k}{\sqrt{2\omega}} \left[e^{i\mathbf{k}\mathbf{x}} c_{\mathbf{k}}^{(-)}(t) + e^{-i\mathbf{k}\mathbf{x}} c_{\mathbf{k}}^{(+)}(t) \right], \tag{3}$$

where ω is the single-particle energy and $v = (a_1 a_2 a_3)^{1/3}$. The choice of operators $c_{\mathbf{k}}^{(\pm)}(t)$ is dictated by the requirement that the instantaneous Hamiltonian (constructed with the energy-momentum metric tensor) be diagonal for all t (Ref. 4):

$$H(t) = \frac{1}{2} \int d^3k\, \omega_{\mathbf{k}}(t) \left[c_{\mathbf{k}}^{(+)}(t) c_{\mathbf{k}}^{(-)}(t) + c_{\mathbf{k}}^{(-)}(t) c_{\mathbf{k}}^{(+)}(t) \right],$$

where, as is easily seen,

$$\omega_{\mathbf{k}}^2(t) = \mathbf{p}^2(t) + m^2 - Q(t), \tag{4}$$

$p_\alpha(t) = \mathbf{k}_\alpha / a_\alpha(t)$ being the components of the physical momentum.

The time evolution of the operators $c_{\mathbf{k}}^{(\pm)}(t)$ can be represented by the time-dependent Bogolyubov transformation

$$c_{\mathbf{k}}^{(-)}(t) = a_{\mathbf{k}}(t) c_{\mathbf{k}}^{(-)}(-\infty) + \beta_{\mathbf{k}}(t) c_{-\mathbf{k}}^{(+)}(-\infty)$$

(it is assumed that $a_\alpha \to$const, $Q \to 0$ for $t \to -\infty$).

The condition that the Hamiltonian be diagonal is satisfied for

$$\beta_{\mathbf{k}}(t) = \frac{i}{2} \sqrt{\frac{v}{\omega_{\mathbf{k}}}} (\dot{g}_{\mathbf{k}} + i\omega_{\mathbf{k}} g_{\mathbf{k}}), \tag{5}$$

where $g_{\mathbf{k}}(t)$ is a solution of the equation

$$\frac{1}{v} \frac{d}{dt}\left(v \frac{dg_{\mathbf{k}}}{dt} \right) + (\omega_{\mathbf{k}}^2 + 2Q) g_{\mathbf{k}} = 0,$$

which is obtained from the Klein-Gordon-Fock equation after separation of the spatial variables; $g_{\mathbf{k}}(t)$ behaves as $(\omega_{\mathbf{k}} v)^{-1/2} \exp(i\omega_{\mathbf{k}} t)$ as $t \to -\infty$.

We examine a state which for $t \to -\infty$ is the vacuum state: $|c_{\mathbf{k}}^{(-)}(-\infty)|0\rangle = 0$.

The number of quasiparticles in mode $\mathbf{k}$ at time t is $n_{\mathbf{k}} = |\beta_{\mathbf{k}}(t)|^2$. It is clear from Eq. (4) that when the anisotropy parameter (2) reaches a threshold value $Q^* = m^2$, the energy of the quasiparticles with $\mathbf{p} = 0$ goes to zero. When this happens, according to Eq. (5) $n_{\mathbf{k}=0} \rightarrow \infty$, that is, a Bose condensate is formed. This is analogous to the phenomenon of pion condensation in a strong external field.[5] The energy of the quasiparticles for $Q = m^2$ has a linear dependence on the momentum $\omega_{\mathbf{k}} = \mathbf{p}$, so this effect can be interpreted as a phase transition of the vacuum $|0\rangle$ into a "superfluid" state.

Near the threshold, the operators $c_0^{(\pm)}$ can be considered c numbers. Along with these, we introduce the canonical variables

$$q = (c_0^{(+)} + c_0^{(-)})/\sqrt{2\omega_0}, \quad p = i(c_0^{(+)} - c_0^{(-)})\sqrt{\omega_0/2}, \tag{6}$$

in terms of which the energy of the $\mathbf{k} = 0$ mode assumes the form

$$H_0(t) = \frac{1}{2}(p^2 + \omega_0^2 q^2), \tag{7}$$

In order to have a consistent description of the situation above the threshold, it is necessary to take into account[5] the self-action of the $\lambda\phi^4$ field. According to Eqs. (3) and (6), the c-number part of the field operator, which corresponds to the condensate mode, can be written as $\phi_0 = qv^{-3/2}$. Then the Hamiltonian of the condensate assumes the form

$$H_c(t) = \frac{1}{2}p^2 + V(q), \qquad V(q) = \frac{1}{2}\omega_0^2 q^2 + \lambda v^{-3} q^4.$$

Above the threshold one has $\omega_0^2 = m^2 - Q(t) < 0$, and the value $q = 0$ is unstable; the stable values of q are those for which $V(q)$ is minimum:

$$q^* = \pm \frac{v^{3/2}|\omega_0|}{2\sqrt{\lambda}}.$$

Since the first term is H_c can be neglected near the threshold (cf. Eq. (7)), the Lagrangian density of the condensate field is of the form $L_c = -v^{-3}V(q^*)$. The energy-momentum tensor of the condensate is in this case

$$T_{ik}^c = V(q^*)\, v^{-3} g_{ik} = -\frac{\omega_0^4}{16\lambda} g_{ik}. \tag{8}$$

We note that this energy-momentum tensor has a vacuum-like form. Since ω_0 depends on t, the tensor T_{ik}^c is nonconservative, a fact which is explained by the transfer of energy from the condensate modes to the condensate. (cf. the analogous situation in Ref. 6). Only the total energy-momentum tensor of the quantized field is conservative.

For the Kasner metric $[a_\alpha(t) \sim t^{p_\alpha}]$ the anisotropy parameter is $Q = 1/9t^2$. It is clear that the onset of the phase transition is at $t \sim m^{-1}$. In this case the energy density of the condensate, according to Eq. (8), is $\epsilon \sim -1/\lambda t^4$, i.e., is of the same order of magnitude as the vacuum polarization.[1,2] At the same time, the spontaneous symmetry-breaking effect due to the self-action in an isotropic metric leads to an energy density $\epsilon \sim -1/\lambda a^4$, which behaves as t^{-2} for a radiation-dominated background.[7]

Thus, in an anisotropic metric at $t \sim t_{pe} = \sqrt{G}$, the energy density of the condensate can have a substantial effect on the evolution of the metric and, in particular, can lead to the removal of the singularity (in this regard it is of interest to construct self-consistent models without singularities, analogous to those found for the isotropic case in Ref. 8).

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