Anderson localization in a flow structure

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The dependence of mobility threshold in macroscopic flow structure on its proximity to the geometrical flow threshold is determined. The localization length in the two-dimensional cases and also the anomalous temperature dependence of the conductivity are estimated. The influence of a magnetic field on the mobility threshold and the negative magnetoresistance are discussed.

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1. Considerable progress has recently been made in understanding Anderson localization,[1–3] primarily because of the ability to calculate the corrections in the classical Drude equation, which arise due to quantum interference. In this paper this calculation is used to determine the mobility threshold of a metal that forms a macroscopic flow structure. There are two dimensionless parameters in such a problem: x represents the density of broken bonds and y denotes the dimensionless resistance of an elementary link of the percolation network, measured in units of $\hbar/e^2$. The "phase diagram" in the (x,y) plane is shown in Fig. 1. For $x > x_c$ the flow structure is broken and there is no conduction at any temperature (a classical CD dielectric). For $x < x_c$ and a finite temperature, the conductivity is not equal to zero, but at $T = 0$, even if there are no breaks, there is a threshold value $y_c(0)$, such that for $y > y_c(0)$ the electronic states are localized at the Fermi level and the conduction is equal to zero. The solid line $y_c(x)$ in Fig. 1, separates the metal region M and the Anderson dielectric AD, which is nonconducting at $T = 0$ but has an exponetially small conductivity at a finite temperature.

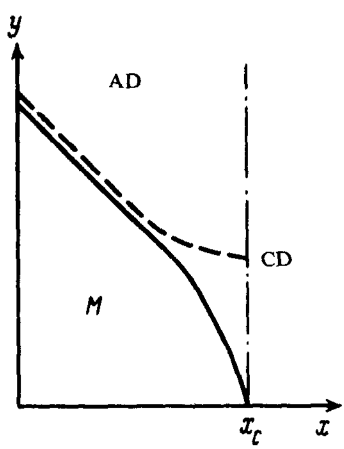

FIG. 1.

Our problem is to find the law describing how $y_c(x)$ goes to zero as $x \to x_c$. The idea is to calculate the quantum correction $\delta\sigma$ for the conductivity. At the mobility threshold this correction is of the same order of magnitude as the classical conductivity. Thus, if we know the dependence of all the values contained in the expression for $\delta\sigma/\sigma$ on $x_c - x$ and y, we can determine the $y_c(x)$ dependence.

2. The expression for the relative correction for the conductivity has the form[3]

$$\frac{\delta\sigma}{\sigma} = -\frac{1}{2\pi^2}\int \frac{d\mathbf{q}}{D(q)\,\mathbf{q}^2}\,. \tag{1}$$

In Eq. (1) $D(q)$ is the effective diffusion coefficient, which is inversely proportional to y and depends on $x_c - x$. In addition, $D(q)$ has a severe spatial dispersion for $qL_c \gg 1$, where $L_c(x) \sim (x_c - x)^{-\nu}$ is the correlation length of an infinite cluster and $\nu = 0.8$ is the corresponding critical index

$$D(q) = \frac{1}{y}(x_c - x)^t f(qL_c), \tag{2}$$

$$f(z) = \begin{cases} 1, & z \ll 1 \\ z^\gamma & z \gg 1 \qquad \gamma = t/\gamma > 1\,. \end{cases} \tag{3}$$

Here $t = 1.5$ is the critical conductivity index in the flow problem. The index y can be determined if $D(q)$ is independent of $x_c - x$ when $qL_c \gg 1$. After substituting Eqs. (2) and (3) in Eq. (1), we see that the integral is determined by the region $qL_c \sim 1$. At the mobility threshold, when $(\delta\sigma/\sigma) \sim 1$,

$$y_c(x) \sim (x_c - x)^{t-\nu}. \tag{4}$$

3. If $y > y_c$ or the system is two-dimensional, then the electronic states will be localized at the Fermi level. However, such localization can be perceived in a specimen of finite size only if it is larger than the characteristic localization length L_{loc}. The localization length, which is also included in the dielectric constant at $T = 0$, is generally the most important characteristic of the localized states. This length can be estimated because the effective resistance for a sample of such dimensions is of the order of $\hbar/e^2$, and hence the relative correction for the conductivity is of the order of unity. Therefore,

$$L_{loc} \sim l_c\left(\frac{y_c}{y}\right)^{\nu/t} \sim y^{-\nu/t}(x_c - x)^{-\nu^2/t}. \tag{5}$$

In the two-dimensional problem

$$L_{loc} \sim \begin{cases} L_c \exp\left\{a\,\dfrac{(x^c - x)^t}{y}\right\}; & a \sim 1;\; y(x_c - x)^{-t} \ll 1 \\ y^{-\nu/t}; & y(x_c - x)^{-t} \gg 1 \end{cases} \tag{6}$$

4. At a finite temperature all of these effects are masked by inelastic processes.[2] At sufficiently high temperatures, when L_{loc} and L_c exceed the diffusion length $L_{in} \sim (D(1/L_{in})\tau_{in})^{1/2} \sim \tau_{in}^{\nu/(2\nu+t)}$, the localized effects are manifested in an anomalous correction for the conductivity that increases with decreasing temperature

$$\frac{\delta\sigma}{\sigma} \sim - L_{in}^{t/\nu} \sim -\tau^{t/(2\nu+t)} . \tag{7}$$

5. The physical meaning of the results obtained lies in the fact that, although the scattering by an individual impurity is relatively weak, the system geometry is such–the macroscopic diffusion is so weak–that the scattered wave is very effectively returned to the original scattering center and is scattered multiply by it, which enhances the localization effects. In this case the principle advanced by Thouless (see, for example, Ref. 4 and also Ref. 1) that the macroscopic resistance is the determining factor, is satisfied. This principle, however, is not universal. The fact is that the returning wave must be coherent with the wave that was originally scattered by the impurity. In an external magnetic field[5,6] the returning wave is not coherent with the incident wave, because the phase change along the classical trajectory is proportional to the magnetic flux that passes through the area stretched along this trajectory. (It is clear that the magnetic field effects are missing in a structure like wood without any rings.) Thus, if a magnetic flux greater than $\phi_0 = \pi\hbar c/e$ passes through an area with linear dimensions of the order of L_c, then the increase in resistance, which occurs as the flow threshold is approached $x \to x_c$, will cause no changes in the mobility threshold. As $x \to x_c, y_c(H,x) \to y_c(H,x_c)$

$$y_c(H, x_c) \sim L_H^{1-\gamma} \sim \left(\frac{eH}{\hbar c}\right)^{(t-\nu)/2\nu}, \tag{8}$$

where

$$L_H \sim \sqrt{\phi_0/H} \sim \sqrt{\hbar c/eH} .$$

The distances greater than L_c are important in a weak magnetic field when $HL_c^2 \ll \phi_0$. In this case[5,6] the integral on the right-hand side of Eq. (1) must be written in the form

$$\frac{4eH}{\hbar c} \sum_n \int \frac{dq_z}{Dq_z + \frac{4eHD}{\hbar c}\left(n + \frac{1}{2}\right)} .$$

As a result, $y_c(x)$ is displaced toward larger y

$$y_c(x, H) - y_c(x, 0) \sim \sqrt{H} . \tag{9}$$

The dashed line in Fig. 1 represents the $y_c(x,H)$ dependence. A negative magnetoresistance should be observed in the metallic phase

$$\left(\frac{\delta\sigma}{\sigma}\right)_H - \left(\frac{\delta\sigma}{\sigma}\right)_{H=0} \sim \begin{cases} H^{(t-\nu)/2\nu}; & L_H >> L_c \\ \sqrt{H} ; & L_H << L_c \end{cases} . \tag{10}$$

6. The flow structure can be realized by various methods, including the use of powders of fine metallic particles baked in a solid dielectric. In dealing with real systems we should remember that the whole analysis was performed by using one-electron approximation and that the problem of mobility threshold for interacting electrons has a number of important peculiarities (see, for example, Ref. 5, 7, and 8) and requires additional study. It is important to note that the Aronov-Altshuler effect,[7] which is enhanced as $x \to x_c$, does not vanish in an external magnetic field.

In conclusion, we note that the analyzed problem resembles in its results the problem of the transition temperature in a Heisenberg ferromagnet whose atoms form a flow structure.[9] This indicates that there is a relationship between the mobility threshold problem and the theory of phase transitions.[11,10]

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[1]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[2]P. W. Anderson, E. Abrahams, and T. V. Ramakrishnan, Phys. Rev. Lett. **43**, 718 (1979).
[3]L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Pis'mi Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].
[4]D. J. Thouless, Phys. Rev. Lett. **39**, 1167 (1977).
[5]B. L. Altshuler, D. E. Khmel'nitzkiĭ, A. I. Larkin, and P. A. Lee, To be published.
[6]S. Hikami, A. I. Larkin, and Y. Nagaoko, Prog. Theor. Phys. To be published.
[7]B. L. Altshuler and A. G. Aronov, Solid State Commun. **36**, 115 (1979); B. L. Altshuler, A. G. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**, 1288 (1980).
[8]L. Eleishman and P. W. Anderson, Phys. Rev. **B21**, 2366 (1980).
[9]E. F. Shender, Zh. Eksp. Teor. Fiz. **70**, 2251 (1976) [Sov. Phys. JETP **43**, 1174 (1976)].
[10]F. Wegner, Z. Phys. **B35**, 207 (1979).
[11]K. B. Efetov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. **79**, No. 9 (1980) [Sov. Phys. JETP].