Critical electric fields and currents of two-dimensional systems

S. E. Burkov and V. L. Pokrovskii

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

(Submitted 23 June 1980)

Pis'ma Zh. Eksp. Teor. Fiz. **32**, No. 4, 285–290 (20 August 1980)

The dependence of the critical current in a modulated-thickness superconducting film on the transverse magnetic field has been found. An analogous effect—the critical longitudinal electric field—for an electron lattice on a helium surface is pointed out.

PACS numbers: 67.20. + k

Recently considerable progress has been made in the study of thin variable-thickness superconducting films. In the experiments of Martinoli *et al.*[1] an aluminum film was investigated that had a one-dimensional periodic corrugation. The film was placed in a magnetic field perpendicular to its surface. In a smooth film a triangular lattice of Abrikosov eddies would appear, the period a of which is specified by the field, so that a quantum of flux Φ_0 is present in a unit cell. On the other hand, it is advantageous to arrange the eddies at the thin points of the film since the eddy energy is proportional to its length. In a commensurate phase (when all eddies lie in valleys) a potential barrier exists, preventing the shifting of the lattice by the modulation period a_0, and the critical current differs from zero. This system is a rare object, in which the Penning force can be taken into exact account since it is caused by the regular modulation of the film thickness whose parameters (depth, shape, period) are well known. In addition, such a film is described by the theory of two-dimensional systems on a periodic substrate, proposed by Pokrovskii and Talapon.[2] The corrugated film is an object in which the restrictions of this theory best match the experimental situation.

There is one other system, very similar to that described above: a monolayer of electrons on the surface of liquid helium in a transverse electric field. The electrons also form a regular triangular lattice, the period of which is determined by the field. Such a lattice was recently detected in an experiment by Grimes and Adams.[3] A weak electric field, varying in space with one-dimensional periodicity, can also be created, for example, by locating a system of parallel electrodes beneath the helium surface. A longitudinal critical electric field, analogous to the critical current in a superconductor, should exist in this system. Under certain conditions a complete analogy exists between the two systems. Therefore we shall restrict our discussion to an eddy lattice, and at the end of the paper we shall indicate the relationship between the two problems.

We define the initial incommensurability p as a deformation leading to some vector of the reciprocal lattice and $2\pi M/a_0$ coinciding with each other. The quantity p is related to the magnetic field H by the expression (M, n, k are integers):

$$p = \frac{H - H_{Mnk}}{2H_{Mnk}}\ ; \quad H_{Mnk} = \frac{\sqrt{3}}{2}\frac{\phi_0}{a_0^2}\frac{M^2}{n^2 + k^2 + nk}, \tag{1}$$

where H_{Mnk} is the value of the field corresponding to exact commensurability. It was shown in Ref. 2 that critical value p^c_{Mnk} of the quantity p exists which separates regions of commensurable (C) and incommensurable (I) phases. In the I phase a soliton superstructure appears: there are long portions in which the eddies are located in valleys, alternating with narrow strips (solitons) in which $N \pm 1$ eddies are present in N periods of the substrate. The solitons are oriented at a 45° angle to the corrugation.[2] In the I-phase the energy of the system does not change when the lattice of eddies is translated through an arbitrary distance. The appearance of an arbitrarily weak current causes a movement of the solitons, accompanied by dissipation. Therefore the critical current for the I-phase is equal to zero. In the C-phase a finite current is required for shifting the lattice. If edge effects are ignored, then the critical current is determined by the condition that the Lorentz force $j\Phi_0 d/c$ and the maximum Penning force are equal. At even smaller currents, however, the system becomes unstable with respect to the production of solitons at the boundaries.

The system of Abrikosov eddies is decribed by the Hamiltonian ($M = 1$):

$$\mathcal{H} = \int \left[\frac{\mu}{2}\left(\frac{\partial \phi}{\partial x} - \frac{\partial w}{\partial y} - 2p\right)^2 + \frac{\mu}{2}\left(\frac{\partial \phi}{\partial y} + \frac{\partial w}{\partial x}\right)^2 + \widetilde{V}(\phi)\right] dx\, dy \tag{2}$$

ϕ and w are the components of the displacements of the eddies from the equilibrium positions in the C-phase along the x and y axes, respectively.[2] (The current $\mathbf{j}$ flows along the y axis). μ is the modulus of the shift of the eddy lattice. In the London case ($H \ll H_{c2}$) the modulation of the film thickness $d + \Delta d(\phi)$ and the current j lead to the potential $\widetilde{V}(\phi)$:

$$\widetilde{V}(\phi) = V(\phi) - f\phi = \frac{1}{s_0}\left[\Delta d(\phi)\left(\frac{\Phi_0}{4\pi\lambda_L}\right)^2 \ln\frac{\min(a, \lambda_e)}{\xi} - \frac{j\Phi_0 d}{c}\phi\right], \tag{3}$$

where $s_0 = \Phi_0/H$ is the cell area, $\lambda_e = \lambda_L^2/d$. The potential $\widetilde{V}(\phi)$ is shown in Fig. 1. The compression modulus for eddies in a thin film is infinite; therefore $\dfrac{\partial\phi}{\partial x} + \dfrac{\partial w}{\partial y} = 0$. Commensurable phase corresponds to the solution $\phi = \phi_0$; $w \equiv 0$, in which all eddies are located at minima of the potential $\widetilde{V}(\phi)$. However, within a narrow layer near the sample boundary the eddies are displaced from the equilibrium positions. For a sufficiently smooth boundary, ϕ and w depend only on the coordinate normal to the boundary. We found the solution of the problem for specified values of $\phi = \phi(\Gamma)$ along

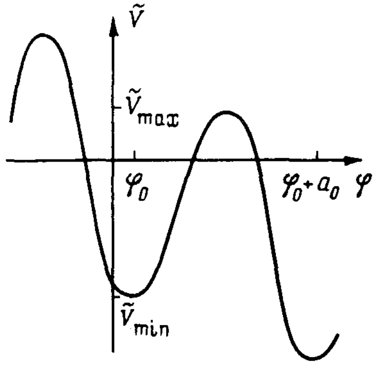

FIG. 1.

the boundary line Γ. The energy, corresponding to this solution, has the form

$$U = \tilde V_{min} S + \oint \left[\int_{\phi_0}^{\phi(\Gamma)} \sqrt{2\left(\tilde V(\phi) - \tilde V_{min}\right)}\, d\phi - h(\theta)\,\phi(\Gamma) \right] dl\, \frac{\sqrt{\mu}}{\sin\theta}, \tag{4}$$

where θ is the angle between the y axis and the normal to the boundary Γ, and

$$h(\theta) = 2p\sqrt{\mu}\,\sin 2\theta. \tag{5}$$

Maximization of the energy (4) with respect to $\phi(\Gamma)$ determines the displacements at the boundary:

$$\tilde V(\phi(\Gamma)) = \tilde V_{min} + \frac{1}{2} h^2(\theta). \tag{6}$$

$h(\theta)$ is maximum for $\theta = \pi/4$. Therefore for $\tilde V_{\max} - \tilde V_{\min} \leqslant \frac{1}{2} h^2(\pi/4)$ the barrier, preventing the production of a soliton in the portion of the boundary with $\theta = 45°$, disappears. The orientation of the produced soliton coincides with the most advantageous orientation of the solitons in the I-phase.[2] Thus, the critical current is determined by the condition

$$\tilde V_{max} - \tilde V_{min} = 2\mu p^2. \tag{7}$$

This result does not depend on the shape of the boundary. Equation (7) shows that the critical current value, which enters into $\tilde V(\phi)$, is strongly dependent on the compression modulus μ. It also determines the phase diagram of the system (see Ref. 2) in the variables H, T. We calculated the compression modulus for $H \ll H_{c2}$:

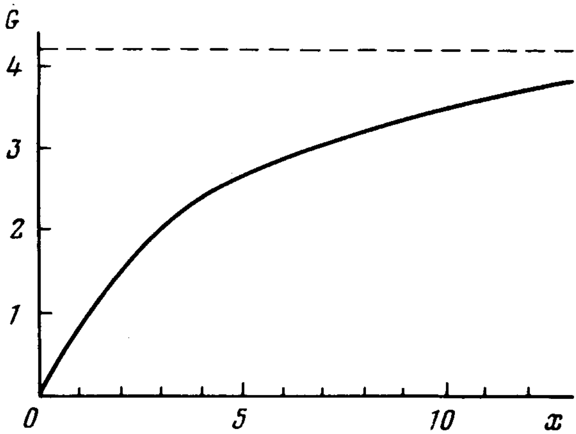

FIG. 2.

$$\mu = \frac{H^2}{32\pi}\left[\frac{s_o}{2\pi\lambda_e} - \sum_q{}' \frac{1+6q\lambda_e}{q(1+2q\lambda_e)^3}\right] = \Phi_o^{1/2} H^{3/2} \frac{12^{1/4}}{2^7\pi^2} G\left(\frac{a}{\lambda_e}\right), \quad (8)$$

where $G(x)$ is a dimensionless function. For $a \gg \lambda_e$, $G = 4.2$ (this limit was obtained in Ref. 5 by another method), for $a \ll \lambda_e$, $G = a/\lambda_e$. Figure 2 shows a graph of the function $G(x)$, obtained by a numerical calculation on a computer. Substitution of (3) and (8) into (7) defines the dependence of the critical current j_c on the initial incommensurability p:

$$\left[\frac{\Delta d(\phi)}{a_o}\frac{\Phi_o}{8\pi\lambda_L^2}\ln\frac{\min(a,\lambda_e)}{\xi} - \frac{j_c d}{c}\phi\right]_{max} - \left[\frac{\Delta d(\phi)}{a_o}\frac{\Phi_o}{8\pi\lambda_L^2}\ln\frac{\min(a,\lambda_e)}{\xi} - \frac{j_c d}{c}\phi\right]_{min} = p^2\frac{H}{16\pi}G\left(\frac{a}{\lambda_e}\right). \quad (9)$$

The graph of $j_c(H)$ for a 1:1 commensurable phase ($M = 1$, $n = 1$, $k = 0$) is shown in Fig. 3. The C-I transition point is defined by the relation[2]:

$$p_c^2 HG\left(\frac{a}{\lambda_e}\right) = \frac{2\Phi_o}{a_o\lambda_L^2}\ln\frac{\min(a,\lambda_e)}{\xi}\left(\frac{1}{2\pi}\int_0^{2\pi}\sqrt{\Delta d(\phi) - \Delta d_{min}}\,d\phi\right)^2, \quad (10)$$

and the maximum possible incommensurability p_{c2} is determined from the condition

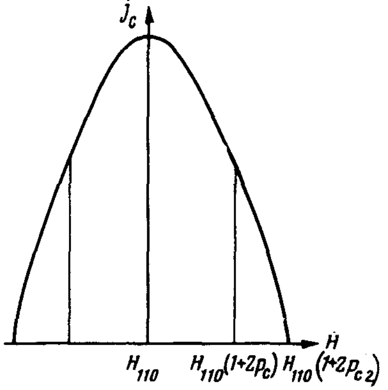

FIG. 3.

$$p_{c2}^2\, H G\left(\frac{a}{\lambda'_e}\right) = \frac{2\Phi_0}{a_0 \lambda_L^2} \ln \frac{\min(a, \lambda'_e)}{\xi} (\Delta d_{max} - \Delta d_{min}). \qquad (11)$$

It is obvious that $p_c < p_{c2}$ always. For $p_c < p < p_{c2}$ the C phase is metastable. Since $j_c = 0$ for the I-phase, the critical current depends on the initial phase of the system, i.e., hysteresis is possible (see Fig. 3). For values $M > 1$ the effective periodic potential $V(\phi)$ decreases[4] as $(\Delta d/d)^M$.

We now turn to the question of the electrons on helium. Their density n is determined by the electric field, perpendicular to the surface: $E_\perp = 4\pi n e$. The interaction between electrons is Coulombic. But the eddies also interact in accordance with the law $(\Phi_0/2\pi)^2 r^{-1}$ for $a \gg \lambda_e$. Finally, the tangential electric field $E_{||}$ produces a constant force $eE_{||}$, acting on the electrons like the Lorentz force $j\Phi_0 d/c$ on the eddies. In the C-phase the maximum possible field $E_{||}^c$ should exist; this is the analog of the critical current of a superconductor. Quantitative results (phase diagram, critical field) for electrons are obtained from the formulas given in this paper and in Ref. 2 for an eddy lattice by using the following "translation rules":

$$\Phi_0 \to 2\pi e\,; \quad H \to \frac{1}{2} E_\perp\,; \quad j \to \frac{c E_{||}}{2\pi d}\,; \quad \lambda'_e \to 0; \quad G\left(\frac{a}{\lambda'_e}\right) = 4{,}2\,. \qquad (12)$$

The periodic potential V for this case depends on the specific modulating field used.

[1]O. Daldini, P. Martinoli, J. L. Olsen, and G. Berner, Phys. Rev. Lett. **32**, 218 (1974).
[2]V. L. Pokrovskii and A. L. Talapov, Zh. Eksp. Teor. Fiz. **78**, 269 (1980) [Sov. Phys. JETP **51**, 134 (1980)]; Phys. Rev. Lett. **42**, 85 (1979).
[3]C. C. Grimes and G. Adams, Phys. Rev. Lett. **42**, 795 (1979).
[4]V. L. Pokrovskii and A. L. Talapov, Zh. Eksp. Teor. Fiz. **75**, 1151 (1978) [Sov. Phys. JETP **48**, 579 (1978)].
[5]K. B. Efetov, Fiz. Tverd. Tela **15**, 647 (1973) [Sov. Phys. Solid State **15**, 459 (1973)].