On electromagnetic showers in crystalline media

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A rapid development of an electromagnetic shower in a crystal is predicted within lengths that are much shorter than the radiation length.

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1. The well-known cascade theory of electromagnetic showers, developed by Bhabha and Heitler,[1] Carlson and Oppenheimer,[2] and Landau and Rumer,[3] refers to the case in which the substance, in which the shower develops, is amorphous. In this case the probability $\pi(\epsilon,\omega)d\omega$ of the emission of a photon with frequency ω by an electron with energy ϵ and the probability $\gamma(\omega,\epsilon)d\epsilon$ of the formation of an electron-positron pair, per unit length, are determined by the known Bethe-Heitler formulas.[3] In this case, according to the equations of the cascade theory, a shower develops within the radiation length L.

In this paper we wish to focus attention on a new important feature of shower development in a substance, involving the fact that in a crystalline substance a shower can develop within distances much shorter than the radiation length L because of coherence effects.

2. A coherence effect in the emission of photons and in the formation of electron-positron pairs in a crystal arises in the case in which the incident particles arrive at the crystal at a sufficiently small angle θ to one of the crystallographic axes. As a result of this effect the cross sections of electromagnetic processes in the crystal are considerably different from the corresponding cross sections in an amorphous medium.

Let us consider, for simplicity, the development of a shower in a crystal in the case in which the shower is formed by a relativistic electron, entering the crystal at an angle θ, and satisfying the condition $2Ze^2/m\theta d \ll 1$, where $Z|e|$ is the charge of the atomic nucleus, d is the distance between atoms in a chain and m is the electron mass. Within this region of angle θ the quantities $\pi(\epsilon,\omega)$ and $\gamma(\omega,\epsilon)$ are determined by the probabilities $\pi_c(\epsilon,\omega)$ and $\gamma_c(\omega,\epsilon)$ of coherent emission and coherent formation of pairs in the crystal.[4,5] At sufficiently high particle energies, when the sizes of the formation zones for bremsstrahlung quanta $l_\gamma = 2\epsilon(\epsilon-\omega)/m^2\omega$ and electron-positron pairs $l_\pm = 2\epsilon_-\epsilon_+/m^2\omega$ exceed $2R/\theta$, where R is the screening radius of the atom, ϵ_- and ϵ_+ are the energies of the formed electron and positron, the quantities π_c and γ_c have the form[5,6]

$$\pi_c(\epsilon,\omega) = \frac{\pi}{2I}\frac{R}{\theta d}\pi_{B-H}(\epsilon,\omega),\quad \gamma_c(\omega,\epsilon) = \frac{\pi}{2I}\frac{R}{\theta d}\gamma_{B-H}(\omega,\epsilon) \tag{1}$$

where $I = \ln 183Z^{-1/3}$, and π_{B-H} and γ_{B-H} are the Bethe-Heitler probabilities of photon emission and pair formation. Substituting the probabilities (1) into the cascade equations of the showers, we arrive at the conclusion that the radiation length L, corresponding to the amorphous case, should be replaced by the radiation length $L_c = L(2I\theta d/\pi R)$, for a crystal. Thus, when the conditions $l_\gamma \gg 2R/\theta$ and $l_\pm \gg 2R/\theta$ are satisfied, the shower development in a crystal should occur over a much shorter length than in an amorphous medium.

3. The inequalities l_γ, $l_\pm \gg 2R/\theta$ are satisfied, for example, for $\epsilon \sim \omega \gtrsim 100$ GeV, i.e., at energies typical of cosmic rays.

In cosmic rays the predicted effect can be observed during the interaction of the particles with a crystallite. In this regard we turn our attention to the following fact: if the size r of the crystallite grains is small $r \ll L_c$, then the shower development will occur in the crystallite the same as in an amorphous medium. The fact of the matter is that in the crystallite the equations of the cascade theory should be averaged over the various grain orientations. Since the averaged values of $\bar{\pi}_c$ and $\bar{\gamma}_c$ are the same as the Bethe-Heitler results for $r \ll L_c$,[4] and the numbers of photons and electron-positron pairs formed within the confines of the individual grains are small, no coherent effects will be evident in the development of a shower in the crystallite.

This conclusion is altered considerably if the grain size is comparable with the length L_c. In this case a considerable change in the shower functions can occur within the confines of the individual grains. In this case the coherence effects should lead to the appearance of events consisting of a large number of particles in lengths of the order of L. Thus, in order to observe the predicted effect in cosmic rays it is necessary to perform an experiment with a crystallite having grains whose sizes satisfy the condition $r \gtrsim L_c$.

The anomalously large electromagnetic showers recorded in the 1950s in emulsions exposed to cosmic rays (see Refs. 7,8 and the literature citations therein) and the recently observed shortened cascades of cosmic μ-mesons[9] apparently are examples of this effect.

4. The rapid development of an electromagnetic shower in a crystal is also possible in the region of lower energies of the electrons striking the crystal when the coherent effect occurs only for the emission process of low-energy photons. In this case $\gamma(\omega,\epsilon) = \gamma_{B-H}(\omega,\epsilon)$ within a wide interval of shower particle energies. In this case, if the energy of the incident electron varies as $E(t)$ with the depth of penetration t into the crystal and the crystal thickness T satisfies the condition $T < L$, then it is easy to obtain the following expressions from the cascade theory equation for the number of low-energy photons N_Ω with frequency $\omega \geqslant \Omega$ and the number of electrons and positrons Π_E with energy $\epsilon \geqslant E$ formed in the crystal

$$N_\Omega = \int_\Omega^\infty d\omega \int_0^T dt\, \pi_c(E(t), \omega), \qquad E(0) \gg \Omega \gg m$$

$$\Pi_E = 2\int_E^\infty d\epsilon \int_\epsilon^\infty d\omega \int_0^T dt \int_0^t dt'\, \pi_c(E(t'), \omega)\, \gamma_{B-H}(\omega, \epsilon), \qquad (2)$$

$$E(0) \gg E \gg m.$$

Assuming for estimation purposes that $\gamma_{B-H} \approx 1/\omega L$, and that $\pi_c \approx R/\omega L\theta d$ for $l_\gamma \geqslant 2R/\theta$ and $\pi_c \approx 0$ for $l_\gamma < 2R/\theta$, we find from (2) that $N_{\sim 2m} \approx RT/L\theta d$ and $\Pi_{\sim 2m} \approx RT^2/L^2\theta d$.

The applicability conditions of Eqs. (2) $l_\pm < d$, $E(0) \gg \omega$, $\frac{2Ze^2}{m\theta d} < 1$ are satisfied, for example, when $E(0) = 10$ GeV and $\theta = 10^{-3}$ rad; therefore the predicted rapid shower development effect in a crystal can be observed in present day accelerators during the interaction of electrons with single crystals.

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