On the widths of the $\Sigma \to \Lambda$ conversion of Σ-hypernuclei

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It is shown that the widths of the levels of Σ-hypernuclei can be small ($\lesssim 10$ MeV for the ground state of $\Sigma\, C^{12}$) because of the dynamic suppression of the probability of the $\Sigma \to \Lambda$ transition for final states of the Λ-hyperon with a wavelength comparable to the size of the nucleus.

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The Σ-hypernuclear states were detected for the first time[1] in CERN experiments on the $(\kappa^-, \pi^\mp)$ reaction in ^{7}Li, ^{9}Be, ^{12}C. The widths of the ground and low-excitation levels are small ($\Gamma < 10$ MeV), whereas the traditional estimate of the rate of $\Sigma \to \Lambda$ conversion in the nuclear mother gives values of 50–150 MeV. Taking account of the spin-isospin dependence of the cross section of the $\Sigma N \to \Lambda N$ process makes it possible to explain (within the framework of the nuclear mother theory) the decrease in the width of certain p-shell states of light Σ-hypernuclei.[2] Another favorable attempt to explain the experiment was undertaken in Ref. 3, where the width of the $\Sigma \to \Lambda$ conversion of the hypernucleus was associated with the probability of the $\Sigma N \to \Lambda N$ transition of an isolated ΣN cluster in the nucleus (here, however, the combined influence of the nuclear nucleons on the conversion process was ignored).

In this paper we show that with the account of the interaction of the hyperons with the self-consistent field of the nuclear nucleons in the initial and final states, the widths of the $\Sigma \to \Lambda$ conversion of hypernuclei should be small ($\Gamma \lesssim 10$ MeV). Qualitatively, the suppression effect is associated with the orthogonality of the wave functions of the bound state of Σ and Λ in the continuous spectrum and with a slow variation of the operator of the $\Sigma \to \Lambda$ transition in the nucleus $V_{\Sigma\Lambda}$. Actually, the forces of the Σ-nucleus and Λ-nucleus are similar (the levels of the Σ- and Λ-hypernuclei are shifted by approximately the mass difference of the hyperons, and the additional small shift $\Delta E \sim 3$–4 MeV[1] can be explained by channel coupling effects) and consequently the wave functions of the hyperon nucleus, corresponding to the different energies, are orthogonal. The potential of the $\Sigma \to \Lambda$ conversion (just like the potentials of the interaction of the hyperons with the self-consistent field of the nucleus) is proportional to the nuclear density, since the radius of the forces of the hyperon-nucleons is of the order of the distance between the nucleons in the nucleus. Then in the calculation of the probability amplitude of $\Sigma \to \Lambda$ conversion we are dealing with the matrix element of the slowly varying operator between orthogonal states, and the result acquires a definite smallness.

In the lowest order with respect to the interaction between the Σ-nucleus and Λ-nucleus channels (as will be seen below, the effect should also occur for the exact solution of the coupled channel problem) the shift ΔE and the broadening Γ of the

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hypernuclear level are determined by a mass operator of the following form:

$$\Delta E - i\Gamma/2 = <_{\Sigma}A \mid V_{\Sigma\Lambda} (E_0 - H_{\Lambda} + i0)^{-1} V_{\Lambda\Sigma} \mid _{\Sigma}A>.$$

Here $|_{\Sigma}A\rangle$ is the state of the Σ-hypernucleus with energy E_0 ignoring channel coupling, $V_{\Sigma\Lambda} = V_{\Lambda\Sigma}{}^{+}$ is the interaction between channels, H_{Λ} is the Hamiltonian of the Λ-hyperon—N-nucleon channel.

Let us assume $|E,\alpha\rangle$ is the eigenfunction of the Hamiltonian H_{Λ} corresponding to the state with total energy E (the index α distinguishes states that are degenerate with respect to E). For the Green's function $(E_0 - H_{\Lambda} + i0)^{-1}$ there exists the spectral representation (the contribution of the bound states of the Λ-hyperon is omitted):

$$(E_0 - H_{\Lambda} + i0)^{-1} = \int dE\, d\alpha \frac{|E, \alpha><\alpha, E|}{E_0 - E + i0},$$

by the use of which we obtain

$$\Gamma = 2\pi \int d\alpha\, |<_{\Sigma}A \mid V_{\Sigma\Lambda} \mid E, \alpha>|^2.$$

Let us consider a simple model, assuming that in both the bound state as well as in the continuous spectrum the interaction of the hyperons with the nuclear nucleons can be described by means of a potential that does not depend on the state of the system of nucleons. In this case Γ has the form

$$\Gamma = \int_0^{E_{max}} \gamma(E)\, \sigma_{E_0}(E)\, dE,$$

$$\gamma(E) = 2\sqrt{E}\, |<\psi_{E_0} \mid v_{\Sigma\Lambda} \mid E>|^2.$$

Here $|\psi_{E_0}\rangle$ and $|E\rangle$ are the wave functions of the bound state of the Σ-hyperon and Λ-hyperon in the continuous spectrum with energy E, $v_{\Sigma\Lambda}$ is the potential of the $\Sigma \to \Lambda$ conversion, $\sigma_{E_0}(E)$ is the probability density of the transition of the nucleons into the state with energy $E_0 - E$, normalized by the condition

$$\int_0^{E_{max}} \sigma_{E_0}(E)\, dE = 1,$$

E_{max} is the maximum energy of the Λ-hyperon (ignoring the nuclear losses $E_{max} = m_{\Sigma} - m_{\Lambda} + E_0 \sim 70$ MeV). We calculated the $\gamma(E)$ function for the ground state of the $_{\Sigma}C^{12}$ hypernucleus, using as $|\psi_{E_0}\rangle$ and $|E\rangle$ the wave functions obtained by solving the Schroedinger equation with Woods-Saxon type potentials

$$v_Y = -c\rho(r) \qquad Y = \Lambda, \Sigma$$

$$\rho(r) = (1 + \exp(r - a)/b).$$

$$a = 2.8 \text{ F}, b = 0.6 \text{ F}, c = 25 \text{ MeV}.$$

The conversion potential was chosen in the form $v_{\Sigma\Lambda} = c\rho(r)$. In the one-channel problem with such a potential the binding energy of the ground state is $\epsilon_0 = 9$ MeV. The calculation result is shown in Fig. 1. To find the total width Γ it is necessary to know the function $\sigma_{E_0}(E)$. The question of the possibility of establishing the form of $\sigma(E)$ from experimental data on the the interaction of moderate-energy hadrons with nuclei will be discussed by us in another paper. For estimation purposes here we took several $\sigma(E)$ distributions with average values $\overline{E} = (0.25–0.5)E_{max}$ and we obtained $5 \lesssim \Gamma \leqslant 10$ MeV for the width of the $\Sigma \rightarrow \Lambda$ conversion of the ground state of the $_{\Sigma}C^{12}$ hypernucleus. Returning to the energy dependence $\gamma(E)$ we note that the reduction to zero (for $p_\Lambda = 2\pi/R$, p_Λ is the momentum of the Λ-hyperon, R is the radius of the nucleus) and the smallness of the matrix element of the transition associated with this are the dynamic "release" effect of the bound states (arbitrarily strong channels), known in the literature as the existence of stable states immersed in the continuous spectrum.[4] The fact that this effect is not a consequence of using perturbation theory is easy to show if the potential of the $\Sigma \rightarrow \Lambda$ conversion is chosen in separable form: $v_{\Sigma\Lambda} = \lambda |\xi\rangle\langle\xi|$ and the results of Ref. 5, where an expression is given for the mass operator, are used.

In conclusion we note that in order to understand the mechanism of the hyperon-nuclear interaction and, in particular, the $\Sigma \rightarrow \Lambda$ conversion we need experimental information about the spectra of the Λ-hyperons, formed in the decay of Σ-hypernuclei and the interaction of slow Σ with nuclei.

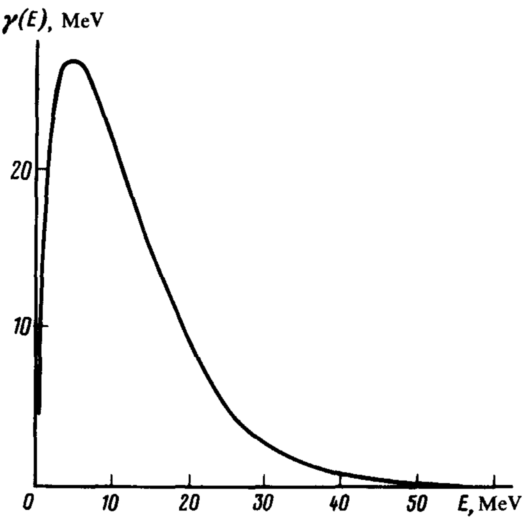

FIG. 1. The $\gamma(E)$ function for the ground state of the $_{\Sigma}C^{12}$ hypernucleus.

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[1]W. Brückner *et al.*, CERN-EP/79–97, 4 September 1979.
[2]C. B. Dover and A. Gal, Phys. Rev. Lett. **44**, 379 (1980).
[3]L. S. Kisslinger, Phys. Rev. Lett. **44**, 968 (1980).
[4]L. Fonda and R. G. Newton, Ann. Phys. (N.Y.) **10**, 490 (1960).
[5]L. N. Bogdanova and V. E. Markushin, Yad. Fiz. **32**, 512 (1980) [Sov. J. Nucl. Phys. **32**, No. 8, (1980 (in press)].