Electrical conductivity of two-dimensional metallic systems

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It is shown that the mechanism of electron-phonon relaxation in a two-dimensional, degenerate electronic system is intrinsically different from that which follows from the Bloch-Peierls theory. This leads to a qualitative difference in the behavior of the electrical resistance of two-dimensional and three-dimensional metallic systems.

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At fairly low temperatures, when the heat pulse of a phonon is small compared with all the characteristic dimensions of the Fermi surface (FS), and the phonon collisions between each other can be disregarded, the distribution function χ of nonequilibrium electrons obeys the equation[1]

$$\operatorname{div} \hat{D} (\nabla \chi - \mathbf{a}\{\nabla \chi\}) = e\mathbf{E}\mathbf{n}, \tag{1}$$

Here, the diffusion tensor $\hat{D}$, which is proportional to the square of the electron-phonon interaction constant, depends on the temperature at T^5, div and ∇ are two-dimensional differential operations in the plane tangential to the FS, $\mathbf{n}$ are unit-vector electron velocities, $\mathbf{E}$ is the electric field strength, and $\mathbf{a}\{\nabla\chi\}$ is an integral linear functional associated with the phonon nonequilibrium. Equation (1) describes the steady-state diffusion of electrons which are continuously "produced" on one-half of the FS (where $\mathbf{E}\cdot\mathbf{n}>0$) and "vanish" on the other half (where $\mathbf{E}\cdot\mathbf{n}<0$). Bloch's law for the temperature dependence of electrical resistance follows from this equation[1]: $\rho_B \sim T^5$.

The situation is totally different when the FS is cylindrical. We can prove rigorously that the integro-differential equation (1) is unsolvable in this and only this case.[2] This request is associated with the peculiarities of a two-dimensional system: the given phonon with a momentum $\mathbf{q}$ can interact with one or several pairs $(\mathbf{p}, -\mathbf{p})$ of symmetric states on the FS (at the point $\mathbf{p}\ \mathbf{q}\cdot\mathbf{n}=0$), but these states in turn interact only with the phonons whose momenta are parallel to $\mathbf{q}$. In this respect, the two-dimensional system differs fundamentally from a three-dimensional system in which any electronic states can exhange phonons between one another. Thus, an electron-phonon system can be broken down into groups between which a momentum transfer is impossible. As the same time, the diffusion flow at a given point $\mathbf{p}$ must have a certain, well-defined direction, and hence the phonons flowing in this direction must be continuously absorbed (or those flowing in the opposite direction must be continuously produced). The relaxation process, therefore, is locally "cut off" at each point on the FS.

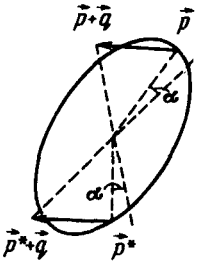

FIG. 1.

We note that this pertains only to a diffusion approximation; a simple division into groups vanishes if the finite phonon momentum is taken into account. However, certain characteristics of the FS geometry in this case are important. In higher orders of expansion over the small parameter q/p_F (p_F is the characteristic size of the FS) the relaxation is possible only if the FS has several pairs of points with parallel normals (this always holds for an open FS). The radii of curvature at these points in this case must be different. Such "superdiffusion," described by a sixth-order differential equation, is characterized by a relaxation time $\tau_{eff} \sim T^{-9}$.

On the other hand, a relaxation can occur irrespective of the structure of the FS, if the finite phonon energy is taken into account. The electronic states. which can interact with a phonon with a given momentum $\mathbf{q}$, in this case are not strictly symmetrical. (An interaction between the centrally symmetric states, as can easily be understood, does not lead to a relaxation of the odd $\chi(\mathbf{p})$ momentum distribution.) As seen in Fig. 1, $\mathbf{p}+\mathbf{q} \neq -\mathbf{p}^*$ and $\mathbf{p}^*+\mathbf{q} \neq -\mathbf{p}$, where the characteristic deflection angle is $\alpha \sim s/v$ (s is the sound velocity and v is the electron velocity). Under these conditions a coupling of the electronic states occurs via diffusion in the phonon momentum space with a characteristic angular step of the order of sv.

Finally, a relaxation mechanism, which is based on relatively infrequent collisions between the phonons, is also possible. By taking these processes into account, we obtain a diffusion equation such as (1), in which the diffusion tensor $\hat{D}$ is proportional to the phonon-phonon collision frequency. The nonlocal nature of the diffusion in this case is associated with the momentum transfer between the phonons that interact with different electrons. The corresponding relaxation time can be represented in the form

$$\tau_{eff} \approx \tau_{pp} \frac{n_e}{n_p}\left(\frac{p_F}{q}\right)^2 \sim T^{-9} .$$

Here, τ_{pp} is the time of the free flight relative to the phonon-phonon collisions, $n_e \sim T/\epsilon_F$ is the number of electrons in the thermal layer, and $n_p \sim T^3$ is the number of phonons. It is clear that $\tau_{pp} n_e/n_p$ represents the time during which an electron is displaced a distance q along the FS as a result of one phonon-phonon collision; the $(p_F/q)^2$ factor represents the number of Brownian steps needed for a relaxation.

A resistance can also be attributed to small deviations of the FS from a truly two-dimensional surface.

The contributions of all the foregoing mechanisms are combined in the following approximate equation:

$$\rho \approx \rho_B \left[\left(\frac{T}{\Theta} \right)^4 + \left(\frac{s}{v} \right)^2 + \gamma^2 \right].$$

Here Θ is the Debye temperature and γ is the characteristic angle of deviation of the FS from the cylindrical shape; the numerical coefficients have been dropped.

It should be pointed out that we obtained the given results by assuming that the phonon spectrum is three-dimensional. If, however, the crystal is two-dimensional in terms of elastic characteristics, then the interaction of electrons with bending vibrations should be taken into account.

[1]At very low temperatures this dependence is replaced by an exponential dependence in metals with closed Fermi surfaces, and a term taking into account the transfer processes should be added in Eq. (1).

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[2]R. N. Gurzhi and A. I. Kopeliovich, Usp. Fiz. Nauk, 1980.

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