Electron thermal conductivity in a tokamak

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An expression was obtained within the limits of strong turbulence for the coefficient of anomalous electron thermal conductivity in a tokamak, which is associated with the development of drift-type oscillations localized in a plasma near the rational surfaces.

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In recent years, many theoretical studies have estimated the coefficient of anomalous electron thermal conductivity χ_e in a tokamak, which is associated with small-scaled, drift-type oscillations localized in a plasma near the rational surfaces.[1–5] The maximum permissible value of χ_e can be estimated in the following way.[5] It is clear that an electron in a wave field cannot be dilodged from its initial location more than $\Delta r \lesssim 1/k_\perp$. This estimate, however, is valid only for short-wave oscillations with $k_\perp \gtrsim \omega_{pe}/c$. As a result of transition to large-scale oscillations, the electron motion will be influenced by a magnetic field that is "frozen into" such oscillations. This field sharply reduces the characteristic mean square of the electron displacement along the radius. The characteristic square of electron displacement along the radius in a wave-packet field is of the order of $(\Delta r)^2 \sim c^2/\omega_{pe}^2$.[5] The characteristic time τ of the phase shift can be determined from the condition that the electrons in a collisionless plasma interact with a wave according to the Landau mechanism, so that $\tau \sim (k_\parallel V_{Te})^{-1}$. The value of $k_\parallel$ can be estimated in the following way: $k_\parallel = \dfrac{m}{r}\dfrac{B_\theta}{B_0} - \dfrac{n}{R}$. Since $m = nq$ on the rational surface, we obtain $k_\parallel \approx \left.\dfrac{dk_\parallel}{dr}\right|_{r_0} \overline{\Delta r} \approx \dfrac{k_\perp S}{qR}\overline{\Delta r}$, where $S = \dfrac{r}{q}\dfrac{dq}{dr}$ is the shear and $\overline{\Delta r} = \bar{r} - r_0$ is the average characteristic deflection with respect to time of an electron from the rational surface. If we assume that $\overline{\Delta r} \sim 1/k_\perp$, the $k_\parallel \approx S/qR$. Using this value of $k_\parallel$, we obtain an equation for χ_e like that of Ohkawa's equation $\chi_e \sim \dfrac{c^2}{\omega_{pe}^2}\dfrac{V_{Te}}{qR}S$.[3,5] We should note, however, that the value $\overline{\Delta r}$ in a highly turbulent mode is comparable to the transverse wavelength only for odd modes in which $\phi(r \to r_0) \sim r - r_0$. In fact, the electron motion along the radius can be described in a drift approximation by the equation

$$\frac{d\Delta r}{dt} \equiv \tilde{V}_r \approx \left\{ i\,\frac{ck_\theta \phi(\Delta r)}{B} + V\frac{B_r(\Delta r)}{B} \right\} e^{-i\omega t + im\theta + in\vartheta}, \qquad (1)$$

where $B_r = ik_\theta A_\parallel$ and $A_\parallel$ is the longitudinal component of the vector potential of a wave. The coupling between ϕ and $A_\parallel$ follows from Maxwell's equation $\Delta A_\parallel = \frac{4\pi}{c} J_\parallel = \frac{\omega^2}{c^2}\epsilon_l (k_\parallel \phi - \frac{\omega}{c} A_\parallel)$, where ϵ_l is the longitudinal component of the dielectric constant tensor of a plasma. For simplicity, we shall examine the long-wave oscillations for which $B_r \to 0$. Thus, it follows from Eq. (1) that the radial displacement of electrons within the limits of strong turbulence $[(ck_\theta \phi / B) \gg \omega / k_\theta]$ can be represented in the following way:

$$\Delta r_{\text{odd}} \approx \frac{2\pi}{k_\perp} (\sin \omega t + 1) \tag{2}$$

for odd $\phi(\Delta r)$ and

$$\Delta r_{\text{even}} \approx \frac{2\pi}{k_\perp} \sin \omega t \tag{3}$$

for even $\phi(\Delta r)$. In the first case $\bar{k}_\parallel \approx S / qR$ and in the second case (even ϕ) $\bar{k}_\parallel \approx 0$, and the higher orders in Δr of the $k_\parallel$ expansion or the toroidal effects must be taken into account in order to obtain the characteristic correlation time. We note that the even oscillation modes, as a rule, are excited first in a plasma.[6]

We assume that the even oscillation modes are excited in the tokamak and estimate the limiting value of the electron thermal conductivity coefficient χ_e in this case. We can obtain the following expression for χ_e from the kinetic equation for electrons[5]:

$$\chi_e \approx \frac{\int d\mathrm{v} \frac{mV^2}{2} \langle \tilde{V}_r(t) \mathrm{Im} \int_0^t \tilde{V}_r^*(t')dt' \rangle f_e}{\frac{3}{2} n_e T_e}, \tag{4}$$

where $\tilde{V}_r$ is determined by Eq. (1) and the angle brackets $\langle\ \rangle$ denote averaging over the statistical ensemble. The integration in Eq. (4) with respect to time must be carried out along the trajectory of electron motion. Therefore, the problem of determining χ_e reduces to determining the correlation function of the radial motion of electrons $K = \{\tilde{V}_r(t) \mathrm{Im} \int_0^t \tilde{V}_r^*(t')dt'\}$. Substituting in K the values of $\tilde{V}_r$ from Eq. (1) and using the relation between $A_\parallel$ and ϕ, we obtain

$$K = \Big\langle \frac{c k_\perp \phi}{B} \left(1 - \frac{k_\parallel v_\parallel}{\omega} \frac{\frac{\omega^2}{c^2}\epsilon_e}{k_\perp^2 + \frac{\omega^2}{c^2}\epsilon_l} \right) \mathrm{Im} \int_0^t dt' \frac{c k_\perp \phi^*(t')}{B}$$
$$\times \left(1 - \frac{k_\parallel(t') v_\parallel}{\omega} \frac{\frac{\omega^2}{c^2}\epsilon_e}{k_\perp^2 + \frac{\omega^2}{c^2}\epsilon_e} \right)^* \tag{5}$$

First, we should point out that the magnetic field B in a tokamak depends on r and θ. Integrating along the trajectory in Eq. (5), we can assume that $\frac{1}{B(t)} \approx \frac{1}{B}\left(1 - \frac{r}{R}\cos\frac{v_\parallel t}{qR}\right)$. Moreover, it appears that

$$\frac{c\, k_\perp \phi(t)}{\bar{B}} \approx \frac{\omega}{k_\perp} \exp\left\{ - i\omega t + i\int_0^t k_\parallel(t')v_\parallel dt' \right\} \tag{6}$$

within the limits of strong trubulence. By using these relations we can rewrite Eq. (5) in the following form:

$$K \approx \sum_{\omega, k}\left(\frac{\omega}{k_\perp}\right)^2 \left| 1 - \frac{\bar{k}_\parallel v_\parallel}{\omega} \frac{\frac{\omega^2}{c^2}\epsilon_l}{k_\perp^2 + \frac{\omega^2}{c^2}\epsilon_\epsilon}\right|^2 \left\{ \delta(\omega - \bar{k}_\parallel v_\parallel) + \frac{r^2}{2R^2}\,\delta\left(\omega - \bar{k}_\parallel v_\parallel - \frac{v_\parallel}{qR}\right) + \frac{r^2}{2R^2}\,\delta\left(\omega - \bar{k}_\parallel v_\parallel + \frac{v_\parallel}{qR}\right)\right\}, \tag{7}$$

where $\bar{k}_\parallel$ is the average value of $k_\parallel$ with respect to time; it is clear that the oscillating part of $k_\parallel$ cannot contribute to the δ function [this follows from relation (6)]. The last two δ functions appear on the right-hand side of Eq. (7) if the toroidal effect is taken into account (the electron motion along the radius in a tokamak is modulated with a modulation frequency equal to the "bounce" frequency, and the modulation depth is proportional to the toroidality $\epsilon = r/R$).

Since we are considering the even oscillations, we can assume that $\bar{k}_\parallel \approx 0$. Substituting Eq. (7) in Eq. (4), we obtain

$$\chi_e \approx \sum_{\omega, k}\left(\frac{\omega}{k_\perp}\right)^2 \left| \frac{k_\perp^2}{k_\perp^2 + \frac{\omega^2}{c^2}\epsilon_l}\right|^2 \frac{r^2}{R^2}\, e^{-(\omega qR/v_{Te})^2}, \tag{8}$$

where $\epsilon_l \approx \omega_{pe}^2/\omega^2$ (for $\omega > \bar{k}_\parallel v_{Te}$).

The general term of the series on the right-hand side of Eq. (7) reaches a maximum in ω when $\omega \approx v_{Te}/qR$, and it reaches a maximum in $k_\perp$ when $k_\perp \approx \omega_{pe}/c$. This is attributable to the aforementioned effects of "freezing" the magnetic field into the long-wave oscillations. After changing over in Eq. (8) from summation to integration with respect to ω and $k_\perp$, we can obtain the value of χ_e with an accuracy to a numerical coefficient:

$$\chi_e \sim \frac{c^2}{\omega_{pe}^2} \frac{v_{Te}}{qR} \frac{r^2}{R^2}. \tag{9}$$

Thus, we can see that the main contribution to the coefficient of anomalous electron thermal conductivity χ_e comes from the toroidal effects as a result of excitation of the even, drift-type oscillation modes in a tokamak, so that $\chi_e \sim \epsilon^2$. Such ϵ-dependence was apparently observed in the recent experiments using the T–11 facility.[7]

[1]B. B. Kadomtsev and O. P. Pogutse, IAEA-CN-37/0-1, p. 649, Vol 1, 1979.
[2]A. B. Rechester and M. N. Rosenbluth, Phys. Rev. Lett. **40**, 38 (1978).
[3]T. Ohkawa, Gen. Atom. Rep. No. JA-AI-4433, May, 1977.
[4]K. Molvig, S. P. Hirshman, and J. S. Whitson, Phys. Rev. Lett. **43**, 582 (1979).
[5]V. V. Parail and O. P. Pogutse, IAEA-CN-38/C-1, 1980.
[6]B. B. Kadomtsev and O. P. Pogutse, Voprosy teorii plazmy (Problems of Plasma Theory), M., Atomizdat, 1967, **5**, p. 235.
[7]V. M. Leonov, V. G. Merezhkin, V. S. Muchovatov, V. V. Sannikov, and G. N. Tilinin, IAEA-CN-38/N-2, 1980.

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