Many-electron holes on the surface of liquid helium

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The possibility for the existence of separate many-electron holes on the surface of liquid helium is suggested. The conditions under which the appearance of such holes is realistic are discussed. The main characteristics of a many-electron hole are described: the binding energy, effective mass and mobility of a hole along the helium surface. The interaction energy of two holes located at a specified distance from each other is calculated.

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One of the possible ways an instability can be generated on a charged surface of liquid helium is by forming separate, many-electron holes. In contrast with the limiting case of a totally charged, free helium surface, when the electric field of a layer of surface electrons compensates for the external field applied above the helium surface and the developed instability leads to the appearance of a crystal structure consisting of many-electron holes,[1] we are interested in the limiting case $n_s \ll n_s^k (n_s^k$ is the maximum density of surface electrons above the helium, $n_s^k \approx 10^9$ cm^{-2}). In other words, we shall examine the densities $n_s \approx 10^6 \times 10^7$ cm^{-2}. The instability of the homogeneous state of a charged helium surface in this case may occur in the region of the applied fields $E_\perp > E_\perp^k$, $E_\perp^k = 2\pi e n_s^k$. The purpose of this paper is to describe certain properties of an individual, many-electron hole within the context of a model assumption that a many-electron system localized in a hole has the form of a charged disk of radius R. This supposition can be justified experimentally.

Within the context of the chosen model, the total energy W associated with a many-electron hole has the form

$$W = \int d^2\mathbf{r}\left\{\frac{\alpha}{2}\left[(\nabla\xi)^2 + \kappa^2\xi^2\right] + QE_\perp n_s(\mathbf{r})\,\xi(\mathbf{r})\right\} + V_c(R) \qquad (1)$$

$$Q = eN \qquad \kappa^2 = \rho g/\alpha .$$

Here Q and N represent the total charge and the number of electrons in the hole, $\xi(\mathbf{r})$ is a self-consistent deformation of the helium surface caused by electron pressure, ρ and α are the density and the surface tension coefficient of liquid helium, g is the acceleration of gravity, κ is the capillary constant (for helium $\kappa \approx 30$ cm^{-1}), $E_\perp$ is the density of the applied electric field, and $n_s(r)$ is the local distribution of electrons along the hole surface. According to the model selected by us,

$$n_s(r) = \frac{1}{\pi R^2}\exp\left(-\frac{r^2}{R^2}\right), \qquad r = |\mathbf{r}|, \qquad (2)$$

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where R is the radius of the electron disk. The value of R, unknown at this time, must be expressed in terms of the total number N of electrons in the hole, in terms of the applied field $E_\perp$, and in terms of the parameters of liquid helium.

The energy v_c (R) is the total Coulomb energy of the electron disk with a uniform density distribution along its surface

$$V_c(R) = \frac{3\pi}{8}\frac{Q^2}{R}. \tag{3}$$

The energy W in Eq. (1) is valid if the following condition is satisfied:

$$\nabla \xi < 1, \tag{4}$$

which corresponds to the limiting case of a sufficiently smooth deformation of the helium surface.

The energy W in Eq. (1) differs from that used in Ref. 2 to formulate a one-electron hole theory by substituting for the zero-point vibration energy of an electron in a self-consistent potential well $eE_\perp\xi(r)$ the Coulomb interaction energy $V_c(R)$, which is the main energy in the case of a many-electron problem. An analogous situation arises, for example, in the calculation of the parameters for a many-electron bubble.[3]

By varying the energy W with respect to ξ, we obtain an equation for mechanical equilibrium

$$\Delta\xi - \kappa^2\xi = a^{-1}QE_\perp n_s(r). \tag{5}$$

A solution of this equation makes it possible to express $\xi(r)$ in terms of Q and R. Substituting in Eq. (1) the solution for $\xi(r)$ of Eq. (5) and performing appropriate integration, we obtain the following final expression for W

$$W = \frac{Q^2E_\perp^2}{8\pi a}\exp\left(\frac{\kappa^2R^2}{2}\right)E_i\left(-\frac{\kappa^2R^2}{2}\right) + V_c(R), \tag{6}$$

where $E_i(x)$ is an integral exponential function. Determination of the radius R can now be reduced to a solution of the equation $\partial W/\partial R = 0$ provided that $\partial^2 W/\partial R^2 > 0$.

We can easily see that a stable solution of R, i.e., a solution of the equation $\partial W = \partial R = 0$ for $W^{\|} > 0$, is possible only in the region $\kappa R > 1$. Thus,

$$R = \frac{3\pi^2 a}{2E_\perp^2}, \qquad \xi(0) \approx -\frac{QE_\perp}{2\pi a}\ln\left(\frac{1}{\kappa R}\right),$$

$$W \approx -\frac{Q^2E_\perp^2}{4\pi a}\left(\ln\frac{1}{\kappa R} - 1\right). \tag{7}$$

According to Eq. (7), the appearance of a hole is energetically advantageous ($W<0$) if $\kappa R < 1/2$. In the expanded form, this inequality reduces to the requirement

$$E_\perp > E_\perp^{min}, \qquad (E_\perp^{min})^4 = 9\pi^4\rho^2g\,a, \qquad E_\perp^{min} \sim 3000 \text{ V/cm}, \tag{8}$$

i.e., to a determination of the minimum applied field $E_\perp^{\min}$, in which an electron system can be localized in a hole. It is interesting to note that the expression for R, which does not depend on the total number N of electrons in the hole, has a scale $R \approx 3 \times 10^{-2}$ cm in the region $E_\perp \sim E_\perp^{\min}$. However, the inequality (4), which can be conveniently rewritten in the form $\xi(0)/R < 1$, is a constraint on the total number of electrons N in the hole for a given intensity of the electric field $E_\perp$

$$\frac{QE_\perp^3}{3\pi^3 a^2} \ln\frac{1}{\kappa R} < 1 . \tag{9}$$

The value of N has a scale $N \approx 10^6$ for a field $E_\perp \sim E^{\min}$.

Using the analogy of one-electron hole theory,[2] we immediately obtain the expression for the apparent effective mass M of a many-electron hole in the case of its motion along the helium surface

$$M = \frac{\rho Q^2 E_\perp^2}{16\kappa a^2} \tag{10}$$

and the expression for the effective mobility μ

$$\mu = 2\sqrt{2}\pi a^2 R/QE_\perp^2 \eta , \tag{11}$$

where η is the coefficient of first viscosity of liquid helium.

It is also of interest to calculate the energy of interaction W_{ij} of two separate holes whose centers are separated from each other a distance $r = |r_i - r_j|$, where r_i and r_j are the coordinates of the centers of the two holes. To calculate the energy W_{ij}, we must substitute in W of Eq. (1) the deformation and the charge distribution for two individual holes which are separated from each other a distance $r > R$ and select the interference terms (an analogous problem arises, for example, in the calculation of the interaction energy of vortex filaments in superconductors).[4] The final experiment for W_{ij} has the following form for two holes of like charge:

$$W_{ij} = -\frac{Q^2 E_\perp^2}{2\pi a} K_0(\kappa r) + \frac{Q^2}{r} , \qquad r = |r_i - r_j| , \tag{12}$$

where $K_0(x)$ is the Bessel function for an imaginary argument. The energy W_{ij} has a minimum at the point $W_{ij}/r = 0$. Therefore, two holes of like charge can exist in a bound, steady state at a finite distance from each other.

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