Multiplicity and correlation distribution in a quark jet

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It is shown that the tunneling mechanism for dissociation of color tubes leads to a Poisson-like multiplicity distribution and a short-range velocity correlation distribution between the hadrons in quark jets.

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It is currently rather popular to support a viewpoint according to which the main fraction of hadrons created in hard[1,2] and soft[3] inelastic single-jet processes (i.e., processes without a production of hard noncollinear gluons) is produced as a result of dissociation of chromoelectric flux tubes which are generated between the quarks and the gluons with large relative momenta. By assuming that the color tubes break up via tunneling of $\bar{q}q$ and gg pairs in the tube,[1,2] we can satisfactorily describe the relative yields and the average transverse momenta of hadrons in single-jet processes.

It was shown recently[3] that an interaction of two quark tubes, which form a gluon tube, can explain a wide Koba-Nilson-Olesen distribution multiplicity distribution in the dominant hh inelastic interactions, even if the distribution due to dissociation of one quark tube is Poisson-like.

We show that the tunneling mechanism for dissociation of a quark tube leads to a narrow multiplicity distribution. We shall examine a simplified, one-dimensional model in which the massless quarks, which break up the tube, are produced with a zero 4-momentum. Suppose that at a time $t = 0$ there exists a tube with length l, which has a large total momentum P. The probability that this tube will exist until the time t_1 without breaking up has the form

$$\exp -(w\, l\, t_1), \tag{1}$$

where the probability w for the production of a $\bar{q}q$ pair in a unit length of the tube per unit time, according to Ref. 1, is

$$w \cong 0{,}08\ \text{GeV}^2. \tag{2}$$

After the initial breakup at a time t_1 the tube will be split into two fragments—a "heavy" fragment (HF) of length l_1, which contains the original, fast quark,[1)] and a "light" fragment (LF) of length $l - l_1$. The average mass of a LF, as shown below, is of the order of the ordinary hadronic masses, but the square of the mass of a HF is large ($\sim P$). A heavy fragment, therefore, is the initiator of a subsequent cascade; this fragment, which can exist a time t_2 with a probability $\exp -(wl_1t_2)$, subsequently splits off a new light fragment, etc. The combined probability density for the occurrence of exactly n dissociations during a time T, where the k th dissociation occurs during the time $(t_k, t_k + dt_k)$ as a result of which the length of the HF is equal to $(l_k, l_k + dt_k)$ has the form

$$dW_n(t_1,\ldots, t_n; l_1,\ldots, l_n)$$
$$= \prod_{k=1}^{n} (w\, dt_k dl_k) \prod_{k=1}^{n+1} \exp -[w l_{k-1}(t_k - t_{k-1})], \quad (3)$$

where by definition $t_0 = 0$, $t_{n+1} = T$, and $l_0 = l$. Integrating Eq. (3) over all t_k and $l_k (k = 1,2,\ldots,n)$ provided that $0 \leqslant t_1 \leqslant \ldots \leqslant t_n \leqslant T$, $0 \leqslant l_n \leqslant \ldots \leqslant l_1 \leqslant l$, we obtain the total probability for n dissociations during a time $T - W_n$, which can be conveniently represented by using the $W(z)$ generating function:

$$W(z) \equiv \sum_{n=0}^{\infty} z^n W_n = (1/2\pi i) \int_{-i\infty}^{i\infty} \frac{d\beta}{(wl - \beta)}$$
$$\times (\beta - wl/\beta)^z = {}_1F_1(1 - z;\ 1;\ -wlT), \quad (4)$$

where ${}_1F_1(a,b;z)$ is a confluent hypergeometric function. At large T we have

$$W(z) = [(wlT)^{z-1}/\Gamma(z)]\,(1 + O(1/wlT), \quad (5)$$

which differs only by the $1/\Gamma(z)$ factors from the Poisson generating distribution function with $\langle n \rangle = \ln(w/T)$.

Differentiating Eq. (5) with respect to z, we obtain

$$\langle n \rangle = W'(1) = \ln(wlT) + 0.577,$$
$$f_2 \equiv \langle n(n-1) \rangle - \langle n \rangle^2 \approx -1.64. \quad (6)$$

Thus, the distribution (5) is narrower than the Poisson distribution with the same $\langle n \rangle$. Figure 1 shows a dependence of f_2 on $\langle n \rangle$, which was calculated by using an exact generating function (5). We can see that f_2 initially decreases almost linearly with increasing $\langle n \rangle$, and then flattens out and approaches a constant value according to Eq.

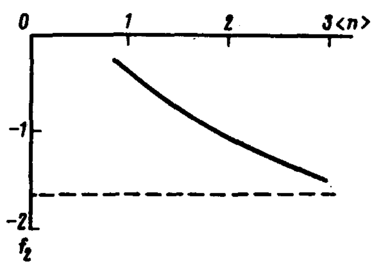

FIG. 1

(6). It is interesting to note that there is experimental evidence for such behavior of f_2 in the processes that form a quark jet.[4] Further experiments in this direction are crucial in selecting a quark-jet-production model.

An inclusive single-particle spectrum for the lifetime τ of HF[2)] can evidently be obtained from the expression

$$f_1(\tau) = \sum_{n=0}^{\infty} \sum_{k=1}^{n+1} \int dW_n \delta[\tau - (t_k - t_{k-1})], \tag{7}$$

where aW_n is given by Eq. (3). Higher-order inclusive spectra can be obtained in a similar manner. By introducing a variable $x = \tau/T$, we obtain (if the exponentially small contributions are ignored for large τ and T)

$$f_1(x)\,dx = dx/x\,,$$

$$f_2(x_1, x_2) - f_1(x_1) f_2(x_2) = dx_1 dx_2 \left[\ln\left(1 + \frac{x_2}{x_1}\right) + (x_1 \leftrightarrow x_2)\right]. \tag{8}$$

Thus we obtain a plateau and short-range velocity correlations (with a correlation length l).

We note that the m^2 distribution in the LF, as can easily be verified, decreases exponentially with increasing m^2, where

$$\langle m^2 \rangle = \rho^2/w \approx 0.5\ \text{GeV}^2 \approx (m_\rho)^2. \tag{9}$$

The LF, therefore, cannot participate in the subsequent cascade; the role of their subsequent disintegration involves an increase of the factor in front of the logarithm in $\langle n \rangle$ apparently by a factor of two to three. We have taken advantage of this above.

Clearly, the cascade will be fully completed when the average mass of a HF is of the order of $\sqrt{m^2}$, i.e., during the time

$$T = P/\rho\,. \tag{10}$$

Thus, the relations (8) give the inclusive spectra of the so-called primary hadrons (in the sense of Ref. 5) in the Feynman x.

In conclusion, we emphasize again the importance of an experimental study of the multiplicity and two-particle-correlation distribution in quark jets.

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[1)]A laboratory reference system, in which the momentum P carries one of the quarks, is selected.
[2)]The lifetime τ of a HF is proportional to the total momentum k of a LF separated from it $k = \rho\tau$, which follows from the main assumptions of the model and of the equations of motion for a quark in a constant field. $\rho \approx 0.2$ GeV^2 is the linear energy density of a tube.

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