Electromagnetic form factors of hadrons at large Q^2 and the confinement effects

A. B. Kaĭdalov
Institute of Theoretical and Experimental Physics

(Submitted 17 August 1980)
Pis'ma Zh. Eksp. Teor. Fiz. **32**, No. 7, 494–498 (5 October 1980)

The influence of effects associated with quark confinement on the behavior of electromagnetic form factors of hadrons at large Q^2 is investigated. Using a topological expansion and color-tube model, it is shown that the Q^2 dependence of form factors is determined by the intersections of the Regge-pole trajectories. The effects associated with the emission of hard gluons are taken into account according to perturbation theory. A good description of the magnetic form of a nucleon is obtained.

PACS numbers: 13.40.Fn, 12.40.Cc

Investigation of the electromagnetic form factors $F(Q^2)$ of hadrons can yield important information on the structure of hadrons and on the interaction of the constituent quarks. The behavior of $F(Q^2)$ at large Q^2 is usually discussed in terms of the quark-counting model[1] in which $F(Q^2) \sim (Q^2)^{1-n}$, where n is the number of elementary constituents of a hadron. The results of quark counting and their logarithmic corrections were recently obtained[2] within the context of quantum chromodynamic (QCD) perturbation theory.

In this paper we emphasize the important role of the effects associated with quark and gluon confinement for $F(Q^2)$ at large Q^2. Although the confinement problem in QCD has not yet been solved, there are nonetheless different confinement models and descriptions of long-range strong interactions that use quarks and gluons. The topological $(1/N_f)$ expansion (TE)[3–5] and the bag and string models[6–8] are among such examples.

We shall discuss the space-time picture of hadron interaction at high energies by using the color-tube model. This corresponds to planar TE diagrams. To identify the planar diagrams, it is convenient to examine a process in which the quantum numbers vary in the t channel (for example, $\pi^+\pi^- \rightarrow \pi^0\pi^0$). Since the slow quarks are annihilated in this coordinate system (henceforth we use the c.m. system), in order for the process to occur the rare quark-parton configurations must be realized in the hadrons when the velocities of q and $\bar{q}$ differ greatly in a single hadron, $-(y_q - y_q) \approx 1/2 \ln s/m^2$. As a result of hadron collision, the slow quarks are annihilated and the fast "spectator" quarks continue to move in the previous direction. The chromoelectric quark field produces a color tube that is divided into two parts due to production of a $q\bar{q}$ pair in a vacuum. As a rule, this process remains in effect, and the most probable configuration contains $\sim y_a - y_b \equiv \xi$ hadrons. The cross section of such process, which is accompanied by annihilation of valence quarks, is $\sigma^{(\text{annih})}_{ab\to x}$ (which corresponds to the difference in the cross sections for physical processes; for example, $\sigma^{(\text{tot})}_{\pi^+\pi} - \sigma^{(\text{tot})}_{\pi^+\pi^+}$) can be written as follows:

$$\sigma^{(\text{annih})}_{ab\to x}(y_a - y_b) = w(y_{q_a} - y_{\bar{q}_a})\, w(y_{q_b} - y_{\bar{q}_b})\, \sigma_{\bar{q}_a q_b} W_{q_a \bar{q}_b \to x}, \tag{1}$$

where $w(y_{q_i} - y_{\bar{q}_i})$ is the probability of finding the quarks in a hadron with a specified value $y_q - y_{\bar{q}} \gg 1$, $\sigma_{\bar{q}_a q_b} \sim 1/m^2$ is the cross section for annihilation of slow quarks, and $W_{\bar{q}_a \bar{q}_b \to x}$ is the probability of converting fast quarks into hadrons, which is generally assumed to be equal to unity. Since the result should be independent of the choice of the coordinate system (i.e., the quantity $y_{\bar{q}_a} \approx y_{q_b}$),

$$w(y_{q_a} - y_{\bar{q}_a})\, w(y_{q_b} - y_{\bar{q}_b}) = C w(y_{q_a} - y_{\bar{q}_b}) \approx C w(y_a - y_b) \tag{2}$$

and hence at $y_q - y_{\bar{q}} \gg 1$,

$$w(y_q - y_{\bar{q}}) = C \exp[-\beta\,(y_q - y_{\bar{q}})]\,. \tag{3}$$

Using the same arguments in the region of the impact parameters **b**, we obtain

$$w(y_q - y_{\bar{q}}, \mathbf{b}_q - \mathbf{b}_{\bar{q}}) = \frac{C \exp[-\beta(y_q - y_{\bar{q}})]}{4\pi a'(y_q - y_{\bar{q}})} \exp\left[-\frac{(\mathbf{b}_q - \mathbf{b}_{\bar{q}})^2}{4a'(y_q - y_{\bar{q}})}\right]. \tag{4}$$

Thus, $\text{Im} f^{(\text{annih})}(\xi, b^2)$ has the usual Regge form, which, after comparing it with the contribution of the secondary Regge poles α_R ($R = \rho, A_2, \omega, f$), we can express the parameters β and α in Eq. (4) in terms of the intersection and the slope of these poles is

$$\beta = 1 - \alpha_R(0); \qquad \alpha' = \alpha'_R(0). \tag{5}$$

The probability of a two-particle final state (for example, $\pi^+\pi^- \to \pi^0\pi^0$), which is proportional to $w(\xi, b^2)$, characterizes that rare process in which the color tube, after splitting into two parts, yields two hadrons in the final state.

The $\pi^+\pi^- \to N\bar{N}$ process differs from the case in question because the qq and $\bar{q}\bar{q}$ diquarks, which produce a nucleon and an antinucleon with a q and $\bar{q}$ that fly apart, are formed as a result of splitting of the color tube. The amplitude of such a process is described at high energy by the exchange of baryon Regge trajectories $\alpha_B(t)$ in the t channel. The probability of splitting of a tube in $N\bar{N}$, therefore, is

$$w_{N\bar{N}} = \frac{\sigma_{ab \to N\bar{N}}}{\sigma^{(annih)}_{ab \to x}} \sim \frac{1}{a'_B \xi} \exp\{[-2(1 - a_B(0)) + (1 - a_R(0))]\xi\}. \quad (6)$$

We shall now examine the e^+e^- annihilation into hadrons for large Q^2. The $q\bar{q}$ pair in this case is produced by a virtual photon in a vacuum. The diverging q and $\bar{q}$ produce a color tube, and the hadrons are produced just like in the hadron-hadron interactions examined above.

Within the context of this approach, the hadron form factors $F(Q^2)$ for large Q^2 are associated with the intersections of the Regge-pole trajectories, since the probability of finding two final-state mesons can be expressed in terms of the $w(Q^2,0)$ function of Eq. (4)

$$\frac{\sigma_{ab \to M\bar{M}}}{\sigma^{(annih)}_{ab \to x}} = \frac{\sigma_{e^+e^- \to M\bar{M}}}{\sigma_{e^+e^- \to x}} \sim w(Q^2, 0); \qquad Q^2 >> m^2. \quad (7)$$

We then obtain one following for the meson form factor [1]

$$F_M(Q^2) \approx \frac{C}{\sqrt{a'_R \ln \frac{Q^2}{Q_0^2}}\left(\frac{Q^2}{Q_0^2}\right)^{\frac{1 - a_R(c)}{2}}}. \quad (8)$$

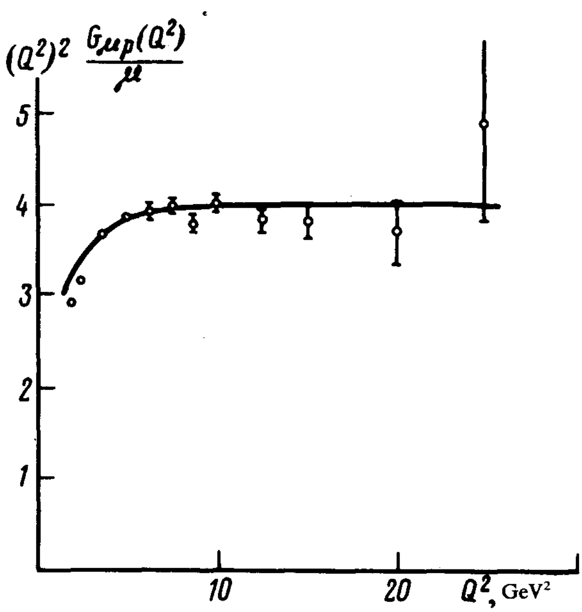

FIG. 1.

Such behavior of $F_M(Q^2)$ differs from that predicted by the quark-counting model[1] and by the QCD calculations based on perturbation theory.[2]

Using Eq. (6), we obtain for the nucleon form factor

$$F_N(Q^2) = \frac{C_1}{\sqrt{\alpha_N' \ln \frac{Q^2}{Q_o^2}} \left(\frac{Q^2}{Q_o^2}\right)^{\frac{1 - 2\alpha_N(o) + \alpha_R(o)}{2}}} . \tag{9}$$

Since the quarks in QCD have a form factor, we shall determine the probability for formation of a collinear $q\bar{q}$ configuration which produces a color tube. It arises when there is no short-range emission of hard gluons with a transverse momentum $p_\perp > Q_1$ in the first stage of the process, where Q_1 is the characteristic transverse momentum of quarks in a hadron. The probability of such an event, which can be calculated from perturbation theory, is determined by the expression

$$S(Q^2) = \exp\left(-\frac{8}{3}\zeta(Q^2)\ln\frac{Q^2}{Q_1^2}\right) = \frac{1}{\left(\frac{Q^2}{Q_1^2}\right)^{8/3\,\zeta(Q^2)}} , \tag{10}$$

where

$$\zeta(Q^2) = \frac{1}{9}\ln\frac{\alpha_S(Q_1^2)}{\alpha_S(Q^2)}; \qquad \alpha_S(Q^2) = \frac{4\pi}{9}\ \frac{1}{\ln(Q^2/\lambda^2)} .$$

Equations (8) and (9) for $F(Q^2)$ must be multiplied by an additional $S(Q^2)$ factor, and the considered mechanism asymptotically leads, as $Q^2 \to \infty$, to a faster decrease of $F(Q^2)$ than the mechanism for the exchange of a minimum number of gluons. We should bear in mind, however, that the value of $S(Q^2)$ varies rather slowly for the existing $Q^2 \lesssim 10^3$ GeV [$8/3\zeta(Q^2) \approx 0.3$–$0.5$], and the considered mechanism may contribute significantly to the hadron form factors. The experimental data for the nucleon form factor are plotted in Fig. 1 on the basis of Eqs. (9) and (10). The parameters λ and Q_1 are equal to 0.3 GeV and 0.5 GeV, respectively, (the result depends weakly on the free parameters). The theoretical curve reproduces the approximate $1/Q^4$ dependence and accurately describes the divergence from this behavior. The form factor of a π meson is also accurately described by this model.

[1)]For the transitions with conservation of helicity [for example, for $F(Q^2)$ of the pseudoscalar mesons], expression (8) must be multiplied by an additional $(Q_0^2/Q^2)^{1/2}$ factor.[9]

[1]S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973); V. A. Matveev, R. M. Muradyan, and A. V. Tavkhelidze, Lett. Nuovo Cim. **7**, 719 (1973).
[2]G. R. Farrar and J. D. Jackson, Phys. Rev. Lett. **43**, 246 (1979); G. P. Lepage and S. J. Brodsky, Phys. Rev. Lett. **43**, 545 (1979); A. F. Efremov and A. V. Radushkin, Preprint JINR E2-11983, 1979.

[3]G. Veneziano, Nucl. Phys. **B74**, 365 (1974); **B117**, 519 (1976).
[4]Chan Hong Mo *et al.*, Nucl. Phys. **B86**, 479 (1975); **B92**, 13 (1975).
[5]G. F. Chew and C. Rosenzweig, Phys. Rev. **41C**, 263 (1978).
[6]A. Casher, J. Kogut, and L. Sussind, Phys. Rev. Lett. **31**, 792 (1973).
[7]A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. **D10**, 2599 (1974).
[8]A. Callan, R. Dashen, and D. J. Gross, Phys. Rev. **D17**, 2717 (1978); **D19**, 1826 (1979).
[9]F. Ravndal, Phys. Lett. **47B**, 67 (1973).

Translated by S. J. Amoretty
Edited by Robert Beyer