Dynamic form factor of superfluid ^{4}He

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The dynamic structure factor of superfluid ^{4}He was observed near absolute zero temperature for frequencies in the range $\tau^{-1} \ll \omega \ll T/\hbar$. An interpretation of the results of the experiments[1] on inelastic scattering of neutrons by superfluid ^{4}He is given.

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In recently published experiments on inelastic scattering of neutrons by liquid ^{4}He, Woods and Svensson[1] measured the temperature dependence (in the range of 1 K to T_λ) of the dynamic form factor $\sigma(\omega,k)$, which can be represented in the form

$$\sigma(\omega, k) = \frac{\rho_s(T)}{\rho}\sigma_s(\omega, k) + \frac{\rho_n(T)}{\rho}\sigma_n(\omega, k). \tag{1}$$

Here $\sigma_n(\omega,k)$ is the dynamic form factor of normal ^{4}He at $T = 2.27\ \text{K} > T_\lambda = 2.18$ K, which for a given k (k varies from 0.8 to 1.96 Å^{-1}), represents a broad velocity distribution of the scattering intensity that is almost temperature independent in the range of T_λ to 4.21 K; $\sigma_s(\lambda,k)$ consists of a narrow, δ-shaped peak corresponding to excitation of phonons and rotons with a Landau spectrum $\omega(k)$ and an addition associated with miltiphonon processes; and $\rho_s(T)$ and $\rho_n(T)$, respectively, are the densities of the superfluid and normal components of superfluid ^{4}He.

The fact that the intensity of the singular part of the microscopic value of $\sigma(\omega,k)$ turned out to be proportional to the hydrodynamic macroscopic value of $\rho_s(T)$ is surprising from the theoretical point of view, although it seems to be experimental proof of a known result obtained by Hohenberg and Martin[2] from qualitative considerations. Griffin[3] also attempted to explain Eq. (1) qualitatively; the singular part of $\sigma(\omega,k)$ was interpreted there as the part produced because of the processes that include the transitions of particles from the condensate to the supercondensate part. As shown by Wong,[4] the interpretation given by Griffin[3] is incorrect, since the intensity of the scattering processes that include the condensate-supercondensate transitions is proportional to the condensate density n_0, rather than to the superfluid density.

The calculation of $\sigma(\omega,k)$ in the quantum region $\omega \gg T/\hbar$, in which the experiment was performed,[1] is impracticable. It is possible, however, to determine $\sigma(\omega,k)$ in the collisionless regime $\omega \gg 1/\tau$, if the condition for adiabaticity of the external perturbation $U(\omega,k)$ is satisfied: $\omega \ll T/\hbar$. The dynamic form factor $\sigma(\omega,k)$ can be expressed in this region in term of the imaginary part of the generalized susceptibility $\alpha(\omega,k)$ (Ref. 5) by the relation

$$\sigma(\omega, k) = \frac{2mT}{\rho\omega}\alpha(\omega, k) \tag{2}$$

where m is the mass of an ^{4}He atom, and the system of equations for determination of the generalized susceptibility consists of a kinetic equation for the distribution function of phonons $n(\mathbf{p},\mathbf{r},t)$, supplemented by the equations for the density ρ and superfluid velocity v_s:

$$\frac{\partial n}{\partial t} - \frac{\partial n}{\partial \mathbf{r}}\frac{\partial H}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}}\frac{\partial H}{\partial \mathbf{r}} = -\frac{n - n_0}{\tau}, \tag{3}$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div}\left(\rho \mathbf{v}_s + \int \mathbf{p}\, n\, d\tau_p\right) = 0, \tag{4}$$

$$\frac{\partial \mathbf{v}_s}{\partial t} + \vec{\nabla}\left(\mu_0 + \frac{U}{m} + \frac{v_s^2}{2} + \int \frac{\partial \epsilon}{\partial \rho} n\, d\tau_p\right) = 0. \tag{5}$$

Here $H = \epsilon(p) + \mathbf{p}\mathbf{v}_s, \epsilon(p) = cp(1 + yp^2)$ is the phonon spectrum, where y depends on pressure, $y > 0$ for $P < 17$ atm, $y < 0$ for $P > 17$ atm, μ_0 is the chemical potential at absolute zero, $d\mu_0 = c^2\frac{d\rho}{\rho}$, and $U(t,\mathbf{r})$ is the external field. Equations (3)–(5), which were used to calculate the temperature correction for the velocity of high-frequency sound (see Refs. 6 and 7), are valid in the phonon temperature region.

A solution of Eqs. (3)–(5) in the (ω,k) representation (see Refs. 6 and 7) gives a linear response $\rho' = \rho - \rho_0$ to the external field $U(\omega,k)$:

$$\rho'(\omega, k) = -m\alpha(\omega, k)\, U(\omega, k), \tag{6}$$

where the generalized susceptibility is

$$\alpha(\omega, k) = \frac{1}{m^2}\,\frac{-(\rho - I_3)}{\left(\frac{\omega}{k} + \frac{\partial c}{\partial \rho} I_2\right)^2 - (\rho - I_3)\left(\frac{c^2}{\rho} - \left(\frac{\partial c}{\partial \rho}\right)^2 I_1\right)}. \tag{7}$$

Here

$$I_1 = \int_0^\infty Z(\omega, k, p)\, dp,$$

$$I_2 = \int_0^\infty \left(\frac{\omega}{kv}\right) Z(\omega, k, p)\, dp,$$

$$I_3 = \int_0^\infty \left(\frac{\omega}{kv}\right)^2 Z(\omega, k, p)\, dp + \rho_n,$$

$$Z(\omega, k, p) = -\frac{\partial n_0}{\partial \epsilon} \frac{p^4}{(2\pi)^2 \hbar^3} \left\{ \frac{\omega}{kv} \ln \left| \frac{\omega - kv}{\omega + kv} \right| + 2 + i\frac{\omega}{kv} [\arctan(kv - \omega\tau) + \arctan(kv + \omega\tau)] \right\},$$

$$v = \frac{\partial \epsilon}{\partial p} = c + 3c\gamma p^2, \quad \rho_n = \frac{2\pi^2 T^4}{45 \hbar^3 c^5},$$

where ρ_n is the phonon part of the normal density of a liquid. Thus, the structure factor is determined by Eqs. (2), (6), and (7). In the neighborhood of the resonances $\omega \approx \pm ck$, assuming that $3|\gamma|T^2c^{-2} \gg (\omega\tau)^{-1}$, we have

$$\sigma(\omega, k) = \frac{T}{mc^2} \frac{\Gamma}{(\omega \mp \tilde{c}k)^2 + \Gamma^2}, \tag{8}$$

$$\tilde{c} = c + \frac{3c\rho_n A}{4\rho} \ln \frac{c^2}{|\gamma| T^2},$$

$$\Gamma = \begin{cases} \dfrac{3\pi\omega\rho_n A}{4\rho}; & \gamma > 0, \\ \dfrac{AT^2}{72 c^3 \rho |\gamma| \tau \hbar^3}; & \gamma < 0 \end{cases},$$

$$A = \left(1 + \frac{\rho}{c} \frac{\partial c}{\partial \rho}\right)^2.$$

The value of Γ determines the attenuation of sound in the high-frequency mode and the time τ in expression (9) for $\gamma < 0$ represents the time of the four-phonon collisions $\tau^{-1} \sim T^7$ (see Ref. 7).

A result analogous to Eq. (7) was recently obtained[8] by using the equations for the correlation functions. However, an elimination[8] in Eq. (7) of the resonance part, which is proportional to ρ_s, is entirely arbitrary. The use of the obtained results (see Ref. 8) for $T > T_\lambda$ also has not been justified, since we go in this case outside the limits of the collisionless region and enter a region in which hydrodynamics can be used (because $\omega < T/\hbar < \tau^{-1}$ for $T > T_\lambda$, where the resonance part is

$$\sigma(\omega, k) = \frac{T}{mc_T^2} \frac{\Gamma_H}{(\omega - ck)^2 + \Gamma_H^2}, \tag{10}$$

where c_T is the isothermal velocity of sound and Γ_H is the hydrodynamic attenuation factor of sound.[5] Thus, we can see from Eq. (8) that the intensity of the resonance part of $\sigma(\omega,k)$, which is $\pi T/mc^2$, does not contain the $\rho_s(T)/\rho$ factor, as predicted in Ref. 2.

The experimental result (1), however, can be explained, in our opinion, by the fact that, whereas $\sigma(\omega,k)$ for $\omega > T\hbar$ varies greatly with the temperature as a result of transition of the λ point, the integral intensity of this quantity—the statistical structure factor[5]

$$\sigma(k) = \int_{-\infty}^{+\infty} \sigma(\omega, k) \frac{d\omega}{2\pi} \qquad (11)$$

is almost independent of the temperature.[9] Of course, it is assumed that the main contribution to the integral (11) comes from the frequencies $\omega \gg T/\hbar$, so that the T/mc^2 factor in Eq. (8) becomes $\frac{\hbar k}{mc}\left(1 - e^{\frac{-\hbar c k}{T}}\right)^{-1} \approx \frac{\hbar k}{mc}$. A small variation of the density-density statistical correlation function $\sigma(k)$ with the temperature is attributable to a small fraction of ^{4}He atoms that precipitate into the condensate $\delta\sigma(k)/\sigma(k) \sim n_0/n$. The proportionality of the resonance part of the scattering intensity of the superfluid component's density can be trivially determined from the temperature independence of $\sigma(k)$:

$$\int_{-\infty}^{+\infty} \left[\sigma(\omega, k) - \frac{\rho_n}{\rho} \sigma_n(\omega, k)\right] \frac{d\omega}{2\pi} \approx \frac{\rho_s(T)}{\rho} \sigma(k) . \qquad (12)$$

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