Amplification of two-dimensional plasma waves in superlattices

A. V. Chaplik
Institute of Semiconductor Physics, USSR Academy of Sciences, Siberian Branch

(Submitted 9 September 1980)
Pis'ma Zh. Eksp. Teor. Fiz. **32**, No. 8, 529–532 (20 October 1980)

If there is a plasma in a spatially periodic potential field, then the transfer processes will play an important role in the damping of plasma waves according to the Landau mechanism. If there is a drift of a two-dimensional, electron plasma relative to a plane superlattice, then the plasma waves can be amplified in a certain interval of drift velocities. It is important that the amplification occurs at much lower drift velocities than the plasmon phase velocity ω/k.

PACS numbers: 72.30. + q

Experimental studies of Langmuir oscillations in two-dimensional electron systems have recently been attracting considerable attention. The characteristic parameters of such systems can be varied within very wide limits, which makes it possible to easily control the wave processes in them. From the viewpoint of application, the question of whether the two-dimensional plasma oscillations can be amplfied is of fundamental interest. As is well known, amplification occurs because of the development of a plasma instability when one part of the plasma medium drifts with respect to another. In the collisionless limit the threshold drift velocity, at which amplification begins, is determined by the Cerenkov criterion, only i.e., it is of the order of the plasmon phase velocity ω/k (ω is the frequency and k is the wave number). The value of ω/k is usually equal to $(2–5)\times 10^8$ cm/sec. the experiments with two-dimensional plasmons.

We show in this communications that amplification is produced in artificial, periodic structures—orientational superlattices—as a result of the drift of a two-dimensional plasma relative to the superlattice (i.e., without separation of the plasma into "quiescent" and "moving" parts). In addition, it turns out that the transfer processes characteristic of waves in the periodic structures reduce the threshold drift velocity. As a result, amplification can occur at drift velocities that are fully attainable by current experimental techniques.

A one-dimensional, orientational superlattice can appear the on the surface of a single crystal whose one Miller index is much larger than the other two.[1] If a metal-insulator-semiconductor (MIS) structure is produced on such a surface and an inversion layer appears, then a two-dimensional plasma of charge carrriers will be in a one-dimensional, periodic potential with a period that is much greater than the constant of the crystal lattice. Typical periods of superlattices produced now are equal to 10^2–10^3 Å.

Dispersion and collisional damping of two-dimensional plasmons in superlattices were, investigated in a paper by Krasheninnikov *et al.*[2] We shall examine the Landau damping and the amplification of oscillations caused by this mechanism because of the drift of a two-dimensional plasma along a superlattice.

We calculate the work of the electric field $E = E_0 \exp i(kx - \omega t)$ of a monochromatic wave, which is done on an electron that moves in a periodic potential $U_0 \cos x/L$. We assume that the amplitude U_0 of the potential is much smaller than the characteristic kinetic energy of the electron. The Fermi energy of electrons is usually an order of magnitude greater than U_0 the experimentally attainable MIS structures with superlattices. In a first approximation with respect to U_0 the law of motion of an electron along the x axis for the initial condition $x(0) = 0$ is given by

$$x(t) = vt + \frac{U_0 L}{mv^2} \sin (vt/L), \tag{1}$$

where v is the time-averaged velocity of above-the-barrier motion of a particle (there can be no trapping in potential wells because of the condition $U_0 \ll mv^2$). The motion is perturbed because of the action of the wave field. This perturbation $\delta x(t)$ in the linear theory in E_0 and in the approximation $U_0 \ll mv^2$ is defined by the equation (compare with Ref. 3)

$$\frac{d^2\delta x}{dt^2} = \frac{eE_0}{m} \exp\left\{ i \left[(kx - \omega - i\gamma)t - (U_0 kL/mv^2) \sin \frac{vt}{L} \right] \right\}, \quad \gamma \to 0. \tag{2}$$

Hence, we obtain

$$\delta x = -\frac{eE_0}{m} \sum_{n=-\infty}^{+\infty} \frac{J_n(a) \exp\{ i [(k + g_n) vt - (\omega + i\gamma) t] \}}{[(k + g_n) v - \omega - i\gamma]^2}, \quad a \equiv U_0 kL/mv^2. \tag{3}$$

Here, $g_n = n/L$ is the one-dimensional, reciprocal-lattice vector and J_n are Bessel functions. The term with $n = 0$ in Eq. (3) corresponds to the spatially uniform case; the remaining terms describe the transfer processes.

The time-averaged energy absorbed by a particle is

$$q(v) = \frac{e}{2} \left\langle \operatorname{Re} \frac{d}{dt} (x + \delta x) E^*(x, t) \right\rangle = -\frac{(eE_0)^2}{2m} \operatorname{Im} \sum_n J_n^2(a) \frac{\omega + i\gamma}{D_n^2}, \tag{4}$$

where $D_n = (k + g_n)v - \omega - i\gamma$. In the derivation of Eq. (4) we must take into two contributions to the particle velocity (proportional to U_0 and E_0) and use the Bessel functions $2nJ_n(z) = z[J_{n+1}(z) + J_{n-1}(z)]$. Going over to the limit $\gamma \to 0$, we obtain for the total absorbed energy Q

$$Q = \int q(v) f(v-u)\,dv; \quad q(v) = \frac{\pi (eE_0)^2}{2m} \sum_n J_n^2(\alpha) \frac{d}{dv}[v\delta(\omega - kv - g_n v)]. \quad (5)$$

Here u is the velocity of the plasma drift relative to the lattice and $f(v)$ is the distribution function of particles on the projection of the velocity in the drift direction. We have for two-dimensional degenerate plasma

$$f(v) = (m/\pi\hbar)^2 \sqrt{v_0^2 - v^2}, \quad v < v_0;$$
$$f(v) = 0, \quad v > v_0, \quad (6)$$

where v_0 is the Fermi velocity equal to $\hbar\sqrt{2\pi N_s}/m$ and N_s is the surface-charge density. Experiments with two-dimensional plasmons are usually performed using silicon MIS structures at helium temperatures in the range of values $N_s \sim 10^{11}$–10^{13} cm^{-2}. Under these conditions the electrons of the inversion layer form a highly degenerate Fermi gas.

As is known, the Landau damping in a spatially uniform Fermi plasma is equal to zero because of the inequality $\omega > kv_0$. As a result, the term with $n = 0$ in the sum of Eq. (5) vanishes. The transfer processes, however, result in a finite Landau damping in a degenerate plasma in a periodic potential. In the experiments k is usually equal to $\sim 10^4$ cm^{-1} i.e., the condition $kL \ll 1$ is satisfied. Thus, the inequality $\alpha \ll 1$ is all the more valid and the terms with $n = \pm 1$ are the principal contribution to the damping Γ. The quantity Γ, which is the imaginary part of the frequency of a plasma wave, is defined as $2\pi kQ/E_0^2$ since the surface energy density of the electric field of a two-dimensional plasmon is equal to $E_0^2/2\pi k$. At $kL \ll 1$ we obtain

$$\Gamma = \frac{e^2 k}{\omega m L^2}\left(\frac{U_0}{\hbar\omega}\right)^2 \left(\frac{4\sqrt{v_0^2 - u_-^2}}{\omega L} - \frac{u_-}{\sqrt{v_0^2 - u_-^2}} + \frac{4\sqrt{v_0^2 - u_+^2}}{\omega L} + \frac{u_+}{\sqrt{v_0^2 - u_+^2}}\right); \quad u_\pm = u \pm \omega L. \quad (7)$$

Equation (7) has meaning for the values of u when all the expressions under the radical are positive. Otherwise, the appropriate terms in Eq. (7) must be dropped. In addition, the inequality $kLU_0 \ll m\omega^2 L^2$ corresponding to the condition $\alpha \ll 1$ is assumed to be satisfied. For the values of N_s and k indicated above $\omega \sim 10^{12}$–10^{13} sec^{-1}, which ensures the satisfaction of this inequality.

The damping, which has a minimum at $\omega \sim v_0/L$ in the absence of drift, increases at small ω and also as $\omega \to v_0/L$

$$\Gamma = \frac{2e^2 k}{m\omega^2 L^3}\left(\frac{U_0}{\hbar\omega}\right)^2 \frac{4v_0^2 - 3\omega^2 L^2}{\sqrt{v_0^2 - \omega^2 L^2}}. \quad (8)$$

Wave amplification ($I < 0$) occurs in the range of u values defined by the equalities

$$\sqrt{v_0^2 + (\omega L/8)^2} + \frac{7}{8}\omega L < u < v_0 + \omega L \qquad (9)$$

[the last two terms in Eq. (7) drop out in this case]. The amplification coefficient increases proportionally to $(u_{max} - u)^{-1/2}$ near the upper limit of the interval $u_{max} = v_0 + \omega L$.

Thus, an amplification can be obtained at drift velocities $u \sim v_0$, which amounts to 3×10^6–3×10^7 cm/sec for typical MIS silicon structures. These rather large drift velocities are, nonetheless experimentally attainable. At the same time, an amplification of two-dimensional plasma waves in a homogeneous system (for example, in an arrangement of two spatially separated layers) requires drift velocities of the order of ω/k. This value can be equal to 10^8–10^9 cm/sec, which is much larger than v_0.

[1]V. A. Petrov, Fiz. Tekh. Poluprovodn. **12**, 380 (1978) [Sov. Phys. Semicond. **12**, 219 (1978)].
[2]M. V. Krashennikov and A. V. Chaplik, Fiz. Tekh. Poluprovodn. **15**, 32 (1981) [Sov. Phys. Semicond. **15**, No. 1 (1981) (in press)].
[3]E. M. Lifshits and L. P. Pitaevskiĭ, Fizicheskaya kinetika (Physical Kinetics), Nauka, Moscow, 1979, 30.

Translated by Eugene R. Heath
Edited by S. J. Amoretty