Nonlinear surface polaritons

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An exact solution is found for the Maxwell equations, which corresponds to a nonlinear surface *H* wave when one of the contiguous dielectric media is optically uniaxial and has a diagonal dielectric constant tensor that depends on the amplitude of the field $\epsilon_{ij}(\omega,|\mathbf{E}|^2)$, $\epsilon_{11} = \epsilon_{22} = \epsilon_0(\omega) + \alpha(|E_1|^2 + |E_2|^2)$, $\epsilon_{33} = \epsilon(\omega)$.

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We show in this paper that allowance for a strong, nonlinear dependence of the dielectric constant tensor of one of the contiguous media gives rise to the appearance of a new type of surface polaritons [nonlinear surface polaritons (*NSP*)]. The structure and existence domain on *NSP* were determined for the boundary of two media, one of which (medium *I*), an isotropic and linear medium with a dielectric constant $\epsilon^I(\omega)$, occupies a half-space $z<0$. With regard to the medium that fills the half-space $z>0$ (medium II), it is assumed that its dielectric tensor $\epsilon_{ij}(\omega,|\mathbf{E}|^2)$ has only diagonal components, among which only the components $\epsilon_{11} = \epsilon_{22} = \epsilon_0(\epsilon) + \alpha(|E_1|^2 + |E_2|^2)$ depend on the field E in the indicated way. We shall show that the Maxwell equations for the surface H wave can be solved exactly under the indicated assumptions. For specificity, we assume that such a wave is propagated along the x axis. We shall seek the field dependence on the coordinates and time in this case in the following form:

$$H_1 = H_3 = 0, \quad E_2 = 0, \quad H_2^{\mathrm{I,II}} = H^{\mathrm{I,II}}(z) e^{-i\omega t + ikx}, \tag{1}$$

$$E_1^{\mathrm{I,II}} = \mathscr{E}_1^{\mathrm{I,II}}(z) e^{-i\omega t + ikx}, \quad E_3^{\mathrm{I,II}} = \mathscr{E}_3^{\mathrm{I,II}}(z) e^{-i\omega t + ikx}.$$

The "ideal" Maxwell equations for the field amplitudes in Eq. (1) have the form

$$\frac{dH^{\mathrm{I,II}}}{dz} = i\frac{\omega}{c} D_1^{\mathrm{I,II}}, \quad kH^{\mathrm{I,II}} = -\frac{\omega}{c} D_3^{\mathrm{I,II}}, \quad \frac{d\mathscr{E}_1^{\mathrm{I,II}}}{dz} - ik\mathscr{E}_3^{\mathrm{I,II}} = i\frac{\omega}{c} H^{\mathrm{I,II}}, \tag{2}$$

where $D_1^I = \epsilon \mathscr{E}_1^I$, $D_3^I = \epsilon \mathscr{E}_3^I$ when $z<0$ and $D_1^{II} = \epsilon_{11}\mathscr{E}_1^{II}$, $D_3^{II} = \epsilon \mathscr{E}_3^{II}$ when $z>0$. Eliminating the quantities $H(z)$ and $\mathscr{E}_3(z)$ from the system of Eqs. (2), we obtain the following equation for the $\mathscr{E}_1(z)$ amplitude:

$$\frac{d^2\mathscr{E}_1}{dz^2} - \tilde{\kappa}^2 \mathscr{E}_1 = 0 \qquad (\text{for, } z<0; \quad \text{here} \quad \mathscr{E}_1 \equiv \mathscr{E}_1^{\mathrm{I}}), \tag{3a}$$

$$\frac{d^2\mathscr{E}_1}{dz^2} - \frac{\kappa^2 \epsilon_{11}}{\epsilon}\mathscr{E}_1 = 0 \qquad (\text{for, } z>0, \quad \text{here} \quad \mathscr{E}_1 \equiv \mathscr{E}_1^{\mathrm{II}}), \tag{3b}$$

where

$$\tilde{\kappa}^2 = k^2 - \frac{\omega^2}{c^2}\epsilon^{\mathrm{I}}, \quad \kappa^2 = k^2 - \frac{\omega^2}{c^2}\epsilon\,. \tag{3c}$$

The solution of Eq. (3a) for $\tilde{\kappa}^2 > 0$ has the usual form for the region $z<0$ (linear medium)

$$\mathscr{E}_1^{\mathrm{I}}(z) = \mathscr{E}_1^{\mathrm{I}}(0)\, e^{\tilde{\kappa} z}, \tag{4}$$

where $\tilde{\kappa} = +\sqrt{k^2 - \frac{\omega^2}{c^2}\epsilon^I}$.

Equation (3b), which describes the field for $z>0$, reduces to the following first-order nonlinear equation:

$$\frac{1}{2}\left(\frac{d\mathcal{E}_1^{II}}{dz}\right)^2 + U(\mathcal{E}_1^{II}) = C\,, \tag{5}$$

where C is a constant and $U(\mathcal{E}_1) = -\frac{\kappa^2}{2\epsilon}\left(\epsilon_0\mathcal{E}_1^2 + \frac{\alpha}{2}\mathcal{E}_1^4\right)$. Since we are interested in solving Eq. (3b), which vanished as $z\to\infty$, the constant $C=0$. In view of this, at ϵ, $\epsilon_0>0$ (we shall bear this case in mind below) the solution of Eq. (5) for $\alpha<0$ gives

$$\mathcal{E}_1^{II}(z) = \sqrt{\frac{2\epsilon_0}{|\alpha|}}\left\{\cosh\left[\left(\frac{\epsilon_0}{\epsilon}\right)^{1/2}(z - z_0)\kappa\right]\right\}^{-1}, \tag{6}$$

where the parameter for the solution of z_0 is determined from the continuity condition for $z=0$ of the tangential component of the electric field intensity:

$$\mathcal{E}_1^{I}(0) = \mathcal{E}_1^{II}(0) = \sqrt{\frac{2\epsilon_0}{|\alpha|}}\left\{\cosh\left[\left(\frac{\epsilon_0}{\epsilon}\right)^{1/2} z_0\,\kappa\right]\right\}^{-1}. \tag{6a}$$

It follows from Eqs. (2) that

$$H^{I}(z) = i\frac{\omega}{c}\,\frac{\epsilon^{I}}{\tilde{\kappa}^2}\,\frac{d\mathcal{E}_1^{I}}{dz}\,, \qquad H^{II}(z) = i\frac{\omega}{c}\,\frac{\epsilon}{\kappa^2}\,\frac{d\mathcal{E}_1^{II}}{dz}\,.$$

Using Eqs. (4) and (6), we can see that the intensity of the magnetic field for $z=0$ is continuous if the following relation is satisfied:

$$\frac{\epsilon^{I}}{\tilde{\kappa}} = \frac{\epsilon}{\kappa}\left(\frac{\epsilon_0}{\epsilon}\right)^{1/2}\tanh\left[\left(\frac{\epsilon_0}{\epsilon}\right)^{1/2} z_0\,\kappa\right], \tag{7}$$

which determines the dispersion law $\omega=\omega\,[k,\mathcal{E}_1(0)]$. It follows from this relation that at $\epsilon^{I}>0$ (specifically, this case corresponds to the boundary between the nonlinear medium and the vacuum) the parameter $z_0>0$; if, however, $\epsilon^{I}<0$, then $z_0<0$. In the first case, the quantity $\mathcal{E}_I^{II^o}(z)$ [see Eq. (6)] increases to a maximum value $\mathcal{E}_{1m} = (2\epsilon_0/|\alpha|)^{1/2}$ as a result of variation of z from $z=0$ to $z=\infty$ (the quantity $\mathcal{E}_{1m}$ is much smaller than the atomic fields only in the frequency region $\omega\approx\omega_{\parallel}$, $\epsilon_0(\omega_{\parallel})=0$ and in the media with fairly large values of $|\alpha|$; otherwise, more general expressions for the ϵ_{ij} and $|\mathbf{E}|$ coupling should be used) and then monotonically decreases to zero with increasing z. In the second case, the field $\mathcal{E}_1^{II}$ monotonically decreases to zero with increasing z as a result of an increase of z from $z=0$ to $z=\infty$.

Taking Eq. (6a) into account, we can represent relation (7) in the following way:

$$\frac{|\epsilon^{I}|}{\tilde{\kappa}} = \frac{\sqrt{\epsilon}}{\kappa}\left(\epsilon_0 + \frac{\alpha}{2}\,\mathscr{E}_1^2(0)\right)^{1/2}$$

or in the form

$$\frac{k^2c^2}{\omega^2} \equiv n^2(\omega) = \epsilon\epsilon_I\,\frac{\epsilon^{I} - \left[\epsilon_0 + \frac{\alpha}{2}\,\mathscr{E}_1^2(0)\right]}{(\epsilon^{I})^2 - \epsilon\left[\epsilon_0 + \frac{\alpha}{2}\,\mathscr{E}_1^2(0)\right]}\,. \tag{8}$$

if Eq. (3c) is taken into account. We can see from the requirement $n^2(\omega) > 0$ that the waves examined above can exist if the value $\varepsilon_1(0)$ satisfies the inequality $\epsilon_0 + \frac{\alpha}{2}\mathscr{E}_1^2(0) > (\epsilon^I)^2/\epsilon$ when $\epsilon^I > \epsilon$ or the inequality $\epsilon_0 + \frac{\alpha}{2}\mathscr{E}_i^2(0) < (\epsilon^I)^2/\epsilon$ when $\epsilon^I < \epsilon$. Since $\epsilon \gg \epsilon$. Since $\epsilon \gg 1$, the last case corresponds to the boundary between the nonlinear medium and the vacuum. It is important that the nonlinear waves under consideration can also exist at $\alpha < 0$ when ϵ, ϵ_0, and $\epsilon^I > 0$; i.e., they can exist in a spectral region in which the linear polaritons do not exist.

In conclusion, we emphasize that the exact solution given above corresponds only to a special form of the $\epsilon_{ij}(\omega, |\mathbf{E}|^2)$ dependence even for a uniaxial crystal II. The presence of unaccounted for and field-dependent components of the ϵ_{ij} tensor does not lead to a noticeable instability of the nonlinear wave when it is converted to body waves only in those cases when these unaccounted for components of the dielectric tensor are sufficiently small.

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