Confinement of monopoles at ultrahigh temperature

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Arguments are presented to show that monopole confinement occurs at temperatures $T \gtrsim 10^2$ GeV. This assumption makes it possible to solve the problem of relict monopoles in unified theories of strong, weak, and electromagnetic interactions.

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1. Unified gauge theories of strong, weak, and electromagnetic interactions, which are based on simple symmetry groups of the type SU (5), SO (10), E_6, etc., the so-called grand unification theories (GUT),[1] have recently acquired ever greater popularity. Recently, however, much doubt has arisen as to how realistic such theories really are. As shown by Zel'dovich and Khlopov[2] and also also by Preskill,[3] many magnetic monopoles with masses of $\sim 10^{16}$ GeV must have been formed in the early universe as a result of phase transition with a breakdown of symmetry between the strong and electroweak interactions at a temperature $T_{c_1} \sim 10^{14}$ GeV.[4,5] It turns out that the density of monopoles, which have not annihilated each other until now, must be inadmissibly large.[2,3]

Numerous attempts to solve the problem of relict monopoles[6] so far have not produced satisfactory result (see discussion of this problem in Refs. 7 and 8), and the initial, somewhat skeptical attitude toward this problem has become a serious fear that the GUT may be incompatible with the modern cosmological concepts.

The purpose of this paper is to point out a simple mechanism, which leads to confinement of magnetic monopoles at an ultrahigh temperatures and to their rapid annihilation in the earliest evolutionary stages of the Universe. This would solve the problem of relict-monopoles. We shall discuss in Para. 2 the mechanism proposed by Nambu[9] for the confinement of monopoles in a superconductor. We discuss in Para. 3 the conditions that are necessary for monopole confinement at high temperature. The monopole-confinement mechanism in GUT is described in Para. 4. The results obtained are discussed in Para. 5.

2. It is known that a magnetic field in a superconductor acquires a mass $m_H = 2e\psi$, where ψ is the density of the Bose condensate of Cooper pairs. Because of this, according to the equation $(\Delta - m_H^2)\mathbf{H} = 0$, the magnetic field can penetrate into the superconductor to a depth of not more than $m_H^{-1} \sim (e\psi)^{-1}$ (Meissner effect). On the other hand, the equation div$\mathbf{H} = 0$ is an identity that is satisfied regardless of the mass in the magnetic field. Therefore, if a monopole is "buried" in a superconductor, then (because of Gauss' theorem) the magnetic flux must be the same at any distance from the monopole. The only possibility of reconciling this condition with the Meissner effect involves the formation of an Abrikosov vortex filament with a thickness $\sim m_H^{-1}$

that contains within itself all the magnetic flux of the monopole. Such a filament can be terminated only at another monopole that is carrying a magnetic charge of opposite sign. Since the filament energy is proportional to its length, a monopole confinement can occur in a superconductor.[9]

3. As one can see from the arguments presented above a *sufficient condition for monopole confinement is the appearance of a mass near the magnetic field in a medium*. Unfortunately, a Bose condensate, which would account for a massive magnetic field, was never present in the universe. In the absence of such a condensate, a mass can appear in the magnetic field only if the polarization operator of the electromagnetic field in the medium is singular at small momenta, $\pi(k) \sim k^{-2}$.[10] In a conventional, low-temperature plasma, which consists of massive charged particles, such behavior of $\pi(k)$ is impossible.[10] At the same time, as mentioned in Ref. 5, because of the exponential infrared divergences that appear in the quantum statistics of massless, non-Abelian fields at $T \neq 0$, the corresponding polarization operator $\pi(k)$ is singular at small k, beginning at e^4, according to perturbation theory. Because of this, the , *non-Abelian* "magnetic" fields can acquire a mass[5] of $m_H \sim e^2 T$ at nonzero temperature (using different arguments, Polyakov[11] predicted a mass $m_H \sim e^2 T$ near non-Abelian "magnetic" fields). As we shall see, this mass is sufficient for monopole confinement in GUT.

4. At a temperature $T \gtrsim T_{c2} \sim 10^2$ GeV, when a symmetry in GUT was re-established to $SU(3)_C \times SU(2)_L \times U(1)_Y$,[4,5] the electromagnetic field represented the sum of a massless, Abelian field B_μ from the $U(1)_Y$ group and a primoridal, massless (ignoring the temperature effects) non-Abelian field A_μ^3 from the $SU(2)_L$ group: monopole was simultaneously a monopole of the massless field $\mathbf{H}_B \text{rot}\, \mathbf{B}$ and of the field $\mathbf{H}_3 = \text{rot}\, \mathbf{A}^3$. This means that if non-Abelian the field H_3, according to Para. 3 acquired a mass $m_{H_3} \sim e^2 T$ at $T > T_{c_2}$, then the filaments of the field H_3 with a thickness $\sim (e^2 T)^{-1}$, which draw the monopoles toward each other with a force that is independent of the distance between them (see Para. 2), had to appeared between the monopoles.

5. As one can see a monopole confinement at $T > T_{c_2}$ in fact occurred if $m_{H_3} \neq 0$. Unfortunately, it is impossible to calculate the exact value of m_{H_3} in terms of perturbation theory, since all the higher orders of perturbation theory contribute $\sim e^2 T$ to this quantity.[5] If, however, the contributions of the higher orders to m^{H_3} vanishes or becomes much smaller than $e^2 T$, then, as shown in Ref. 5, the terms of perturbation theory become divergent for all thermodynamic values. 1). In this case, all the thermodynamic relations, on which the formulation of the problem of the relict monopole is based, will become completely groundless.[5] Thus, the grand unification theory will contradict cosmology[2,3] only if $m_{H_3} \sim e^2 T$ at $T > T_{c_2}$. But in this case, as shown in Para. 4, all the magnetic monopoles will be in the confinement phase at $T > T_{c_2}$. As will soon be shown in a separate paper, a confinement of monopoles leads to their almost instantaneous annihilation, and the density of monopoles that remain after the annihilation is tens of orders of magnitude lower than those densities that contradict the cosmological data.

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1)This is attributable to the presence of the factors $\sim (e^2 T/m_{H_3})N$ in the higher orders of perturbation theory at $T \neq 0$.[5]

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