Asymptotically supersymmetrical model of a single interaction based on E_8

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An asymptotically free model, in which all fermions (as well as bosons) are contained in the fundamental 248 representation, is constructed. [1)] The constructed model is asymptotically supersymmetrical.

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Much attention is now devoted in elementary particle physics to models in which the strong, weak, and electromagnetic interactions are unified within the context of a simple group.[1] The exceptional groups[2] are of great interest. This interest is attributable to several reasons: all these groups contain a certain, naturally identifiable SU(3) subgroup, which is linked with the color group; the E_7 and E_8 groups of relatively small rank must have a sufficiently large, fundamental representation in order to fit all the currently known quarks and leptons into a single, fundamental spinor multiplet. There is a rather large number of models based on the E_6 and E_7 groups[2,3]; however, the models based on the E_8 group have not been noticed until now (see Refs. 4 and 5), although this group has certain, quite remarkable features. It is important that the fundamental representation of E_8 coincide with its adjoint representation. In constructing the models which are asymptotically free relative to all interactions, we must usually add an adjoint multiplet to the fundamental fermion multiplet in order to ensure asymptotic freedom of the scalar fields that can break down the original symmetry to $SU^c(3)\times U(1)$.[3] *The E_8 invariant model seems to be the only one that can have a single fermion multiplet.*

In the model proposed by us, we selected a supersymmetrical set of fields—one 248 multiplet of four-component spinors ψ_a, two 248 multiplets of Higgs M_a and N_a, and one 248 multiplet of the vector fields V_μ^a.

The Lagrangian has the form

$$L = L_g + L_Y + L_s,$$

where the kinetic part L_g is completely defined by the fields and the Yukawa interaction has the form

$$L_Y = -\bar{\psi}_a [\mathrm{p}\,(h_1 M_b + h_3 N_b) + \mathrm{q}\,(h_1^* M_b + h_3^* N_b)]\Gamma^b_{ac}\psi_c, \tag{1}$$

where

$$\mathrm{p} = \frac{1+\gamma_5}{2}, \quad \mathrm{q} = \frac{1-\gamma_5}{2}.$$

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Γ^b_{ac}, the generators of the E_8 group, are normalized by the relation

$$\mathrm{Sp}\,(\Gamma^a, \Gamma^b) = \delta_{ab}\,.$$

The scalar self-interaction, which generally includes the terms that break down the supersymmetry, has the form

$$-L_s = \frac{1}{4}\alpha_1(M^2)^2 + \frac{1}{4}\alpha_2(N^2)^2 + \frac{1}{2}\gamma(MN)^2$$

$$+\frac{1}{2}\delta M^2N^2 + \frac{1}{2}\epsilon(\Gamma^a_{ij}M_iN_j)^2 + \beta_1 M^2(MN) + \beta_2 N^2(MN)\,. \quad (2)$$

In the renormalization-group equations written below, we used the parameters $a_i(i=1,\dots,7)$ instead of the parameters $\alpha_{1,2}, \beta_{1,2}, \gamma, \delta$, and ϵ:

$$a_1 = \gamma;\quad a_2 = 2(\alpha_1 - \gamma - \delta);\quad a_3 = 2(\alpha_2 - \gamma - \delta);$$
$$a_4 = 2\delta - \gamma;\quad a_5 = 2\beta_1;\quad a_6 = 2\beta_2;\quad a_7 = \epsilon\,. \quad (3)$$

The renormalization-group equations in the one-loop approximation of this model have the form:

a) for the gauge coupling constant

$$\dot g = -2g^3 \quad (4)$$

b) for the Yukawa coupling constants

$$\dot h_1 = h_1\left[-6g^2 + 4|h_1|^2 + 2|h_3|^2 + 2h_3^2\frac{h_1^*}{h_1}\right]$$
$$\dot h_3 = h_3\left[-6g^2 + 4|h_3|^2 + 2|h_1|^2 + 2h_1^2\frac{h_3^*}{h_3}\right] \quad (5)$$

c) for the scalar self-interaction constants

$$\dot a_1 = \frac{1}{25}g^4 - \frac{4}{75}\left[\mathrm{Re}\,h_1^2 h_3^{*2} + 2|h_1^2 h_3^2|\right] + 4\,\mathrm{Re}(h_1h_3^*)(a_5 + a_6)$$

$$+ 2a_1[2|h_1|^2 + 2|h_3|^2 - 6g^2] + 520\,a_1^2 + 2a_1(a_2 + a_3)$$

$$+ 12a_1a_4 + 254(a_5^2 + a_6^2) + 4(a_5a_6 + a_1a_7) + \frac{1}{75}a_7^2$$

$$\dot{a}_2 = -\frac{4}{25}\left[|h_1|^4 - 2|h_1^2 h_3^2| - \mathrm{Re}\, h_1^2 h_3^{*2}\right]$$

$$+ 12[|h_1^2| - |h_3^2|]\, a_1 + 4[2|h_1|^2 - 3g^2]a_2 - 4\,\mathrm{Re}(h_1 h_3^*)(a_5 + 3a_6)$$

$$+ 4[|h_1^2| - |h_3^2|]a_4 + 256 a_2^2 + 1280\, a_1 a_2 + 262\, a_2 a_4$$

$$+ 2\, a_7(a_2 + a_3) - 256 a_1 a_3 - 250 a_3 a_4 - 512 a_5 a_6 - 512\, a_6^2$$

$$\dot{a}_4 = 2[2|h_1^2| + 2|h_3^2| - 6g^2]a_4 + 1000 a_1^2 + 504 a_4^2$$

$$+ 250(a_2 + a_3)(a_1 + a_4) + 2000 a_1 a_4 - 250(a_5 - a_6)^2 - 2a_7(8a_1 + a_2 + a_3 + 2a_4)$$

$$\dot{a}_5 = -\frac{4}{25}|h_1^2|\,\mathrm{Re}(h_1 h_3^*) + 2\,\mathrm{Re}(h_1 h_3^*)(6a_1 + a_2 + 2a_4)$$

$$+ (6|h_1^2| + 2|h_3^2| - 12g^2)a_5 + 1280\, a_1 a_5 + 250\, a_4 a_6$$

$$+ 256(a_2 a_5 + a_1 a_6) + 262\, a_4 a_5 + 2a_7(a_5 - a_6)$$

$$\dot{a}_7 = -g^4 - \frac{8}{3}[\mathrm{Re}(h_1^2 h_3^{*2}) - |h_1^2 h_3^2|]$$

$$+ 4(|h_1^2| + |h_3^2| - 3g^2)a_7 + 2a_7(8a_1 + a_2 + a_3 + 6a_4) - \frac{13}{3} a_7^2 \,. \qquad (6)$$

The equations for a_3 and a_6 can be derived from the equations for a_2 and a_5, respectively, by simultaneous substitution

$$h_1 \leftrightarrow h_3, \qquad a_2 \leftrightarrow a_3, \qquad a_5 \leftrightarrow a_6 .$$

Here the dot denotes $16\pi^2 d/dt$.

We seek a solution of Eqs. (5) in the form [6] $h_i = \bar{h}_i g$, where $\bar{h}_i$ are constants. Thus Eq. (5) has two solutions

$$\begin{cases} \bar{h}_i = e^{i\psi} \\ h_3 = \pm i e^{i\psi} \end{cases} \qquad (7) \qquad\qquad \begin{cases} \bar{h}_1 = \cos\phi\, e^{i\psi} \\ \bar{h}_3 = \sin\phi\, e^{i\psi} \end{cases} . \qquad (8)$$

We shall examine only the case (7).

Equations (6) can be solved by using the usual substitution $a_i = \bar{a}_i g^2$, where $\bar{a}_i$ are t-independent constants. All solutions of this type (two one-parameter families and four isolated solutions, among which there is a supersymmetrical solution: $\bar{a}_7 = -1$,

$a_1 = ... = a_6 = 0$) differ from a supersymmetrical solution because of the addition of the terms $\sim 10^{-2}$–10^{-3}. However, these solutions, except the supersymmetrical, give an unstable potential in the Wood approximation. Since this instability is small, it is possible that the radiative corrections can make the effective potential stable and some of these solutions can conceivably give a reasonable, asymptotically free model.

We, however, would like to propose new type of asymptotically free theories—asymptotically supersymmetrical models. It appears that Eqs. (6) have solutions which differ from a supersymmetrical solution by a value that decreases faster than g^2 with increasing energy.

This solution for small g behaves as follows:

$$a_1 \approx 1.57 g_0\, g^{3.99}$$

$$a_4 \approx -3.15 g_0\, g^{3.99}$$

$$a_7 \approx -g^2 - g_0\, g^{3.99}$$

$$a_2 = a_3 = a_5 = a_6 = 0\,,$$

where g_0 is a parameter.

Preliminary estimates show that such solutions can have one more interesting feature: as the energy changes, there is a threshold beyond which the number of massless vector fields increases (i.e., a phase transition occurs).

The structure of an asymptotically supersymmetrical model constructed by us, spontaneous symmetry breaking and its physical consequences will be discussed in a future paper.

[1)]The results of the investigation were reported at the symposium "Quark 80," Sukhumi, April, 1980.

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