Mixing of parton states and the real part of the amplitude of elastic scattering of high energy hadrons by nuclei

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A mixing of parton configurations of a relativistic hadron during its transmission through a nucleus is analyzed in a two-channel model. Allowance for mixing of configurations gives a characteristic, negative, real part of the hadron-nuclear scattering amplitude, which can reach tens of percents of the imaginary part of the amplitude at energies of tens of GeV. This effect is interpreted as an example of inelastic screening in the nuclei.

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The eigenstate method, which was introduced by Pomeranchuk and Feinberg[1] to describe the interaction of relativistic hadrons with the nuclei, has recently been effectively used within the context of the parton model.[2-6] Two complete orthonormalized sets of states are introduced in this method: $|\alpha\rangle$ are the eigenstates of a free Hamiltonian (physical states with the same quantum numbers) and $|k\rangle$ are the eigenstates of the interaction Hamiltonian (states with a specific number of slow partons). These two bases are connected by a unitary rotation matrix $\hat{C}$

$$|\alpha> = \hat{C} \mid k>. \tag{1}$$

Since the scattering-amplitude operator $\hat{f}$ is diagonal in the $|k\rangle$ basis, the diffraction amplitude is

$$f_{\alpha\beta} = \sum_k C_k^\alpha \, (C_k^\beta)^* f_k \,. \tag{2}$$

For brevity, the overlap integral of the high-momentum parts of the parton states $|\alpha\rangle$ and $|\beta\rangle$ has been dropped; this does not affect the subsequent results.

The elastic scattering of hadrons by nuclei was analyzed in the context of the constituent-quark model by Kopeliovich and Lapidus[2] in a two-component approximation in which the passive state without slow partons ($k=0, f_0=0$) and the active state, which includes all the components with slow partons $k \geqslant 1$, are clearly identified. It was found that the difference in the amplitudes f_k with different number $k \neq 0$ can be ignored and that we can set $f_k = f$ for $k \geqslant 1$. This conclusion can be drawn from an analysis of the distribution of partons in the constituent quark,[6] which shows that the parton density is close to saturation at energies attained in the accelerators. The same conclusion can be drawn from quantum-chromodynamic calculations of the pomeron intercept,[7] which greatly exceeds unity. An analysis of the cross sections for interaction of hadrons with nuclei showed[7] that the interaction amplitude of two active quarks is close to the unitary limit.

In a two-component approximation the partial amplitude of interaction of a quark with a nucleus in the optical limit has the form[2]

$$F_{qA}(\mathbf{b}) = i P_q \left[1 - \exp\left(- \operatorname{Im} f \, T(\mathbf{b})\right) \right] , \tag{3}$$

where $\mathbf{b}$ is the impact parameter, $T(b) = \int^\infty \rho(b,l)dl$ is the profile function of the nucleus, and $P_q = \sum_{k=1}^\infty |C_k^q|^2$ is the weight of the active component of a q quark.

We shall now take into account the fact that the active and passive states can interchange during the time $t \approx E/\mu^2$, where μ is the characteristic mass of order 1 GeV. We shall analyze a simplified problem of transmission of a quark through nuclear matter of constant density ρ. The equation, which describes a variation of the wave function of a quark, has the form

$$\frac{\partial \psi}{\partial l} = i \hat{Q} \psi , \tag{4}$$

where ψ is the multicomponent wave function of a quark with the components $\psi_k = |k\rangle$, l is the longitudinal coordinate, and $\hat{Q}$ is the momentum operator. We shall simplify the problem by leaving only two components with $k=0$ and $k=1$. The problem in this case is analogous to the oscillation of K° mesons. The momentum operator has the form

$$\hat{Q} = \begin{pmatrix} q + |C_1|^2 \Delta q & -C_o C_1^* \Delta q \\ -C_o^* C_1 \Delta q & q + |C_o|^2 \Delta q - if \end{pmatrix}. \tag{5}$$

Here $\Delta q = (m_\beta^2 - m_\alpha^2)/2E$ and α and β are hadronic states that are coupled to the $|0\rangle$ and $|1\rangle$ states. In the parton model $1/\Delta q = E/\mu^2$ represents the formation length of parton states.

Solving Eq. (4) with the operator (5), we obtain the following expression for the partial amplitude of the elastic scattering of the α state

$$-iF_\alpha(l) = 1 - \langle \psi_{out}(l) | \psi_{in}(l) \rangle , \tag{6}$$

where $\psi_{\text{out}}(l)$ is the solution of Eq. (4) and $\psi_{\text{in}}(l)$ is the incident wave:

$$-iF_\alpha(l) = 1 - \exp\left(-\frac{fl}{2} - i\Delta q \frac{l}{2}\right)\left[\cos\left(\frac{\lambda l}{2}\right) - \frac{i\Delta q + (2P_\alpha - 1)f}{\lambda}\sin\left(\frac{\lambda l}{2}\right)\right]. \tag{7}$$

Here we denote

$$\lambda = [(\Delta q)^2 - f^2 - 2if\,\Delta q\,(2P_\alpha - 1)]^{1/2}. \tag{8}$$

At high energies, when the mixing can be ignored, we obtain the well-known expression (3) from Eq. (7). At intermediate energies $\Delta q/f \gg 1$ in the limit, i.e., when the mixing of the $|0\rangle$ and $|1\rangle$ states is large, we can expand the expression (7) according to the parameter $f/\Delta q$ and obtain

$$\text{Im}\,F_\alpha(l) \approx 1 - e^{-P_\alpha fl} - \frac{P_\alpha(1 - P_\alpha)f^2}{(\Delta q)^2} e^{-P_\alpha fl} \times [1 - e^{(2P_\alpha - 1)fl}\cos(\Delta ql)]. \tag{9}$$

The first two terms on the right-hand side of Eq. (9) correspond to the standard Glauber-Sitenko approximation in the optical limit. The third term is a correction for the inelastic screening, which was calculated in the first order from the cross section of inelastic diffraction. This correction coincides with the Karmanov-Kondratyuk formula[8] in a two-channel approximation if we assume that the scattering amplitude of the $|\alpha\rangle$ state is equal to that of the $|\beta\rangle$ state, i.e., if $P_\alpha = P_\beta = 0.5$. This can be easily verified if we recall[2] that the cross section of inelastic diffraction of a hadron $|\alpha\rangle$ is $\sigma_{\text{diff}} = P_\alpha(1 - P_\alpha)f^2$.

It follows from expression (7) that the amplitude $F_\alpha(l)$ has a real nonvanishing part whose explicit formula is omitted here for convenience. This contribution to the real part of the amplitude is nonvanishing even if we assume that the quark-nucleon scattering amplitude f is purely imaginary and disregard the contributions to $\text{Re}F_\alpha(l)$ from the secondary Reggeons and from the increase of the total cross sec-

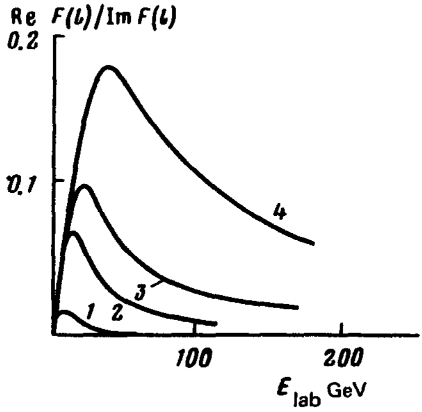

FIG. 1. The ratio ReF(l)/ImF(l) for different values of l. 1, $l = 1f$; 2, $l = 3f$; 3, $l = 5f$; 4, $l = 10f$.

tions of hadron-hadron interactions with the energy.

Figure 1 shows the energy dependence of the value $\epsilon = \mathrm{Re}F_\alpha(l)/\mathrm{Im}F_\alpha(l)$, which was calculated for different values of l, for a constituent quark with the following parameters introduced above: $P_q = 0.6, f = (1/2P_q)\sigma_{qN}\,\rho \cong 0.2\,\mathrm{F}^{-1}$, where $\sigma_{qN} \cong 17$ mb is the total cross section for interaction of a quark with a nucleon and $\rho \cong 0.14\ \mathrm{F}^{-3}$ is the nuclear density of nucleons. The effective mass included in the mixing parameter $\Delta q = \mu^2 E$, which is equal to the average transverse parton mass, is assumed to be $\mu^2 = 1\ \mathrm{GeV}^2$. We can see in Fig. 1 that $\mathrm{Re}F_\alpha(l)$ of the amplitude of the interaction of a quark with a nucleus is negative and its absolute value has a maximum at an energy of several tens of GeV. The location of the maximum depends on the specific choice of μ^2. We can clearly see a transition from the real part of the scattering amplitude of a consitituent quark to a hadron. Thus,

$$\mathrm{Re}\,F_\pi = \mathrm{Re}\,F_q\,(1 - \mathrm{Im}\,F_{\bar q}) + \mathrm{Re}\,F_{\bar q}\,(1 - \mathrm{Im}\,F_q) \tag{10}$$

for a pion consisting of a q quark and a $\bar q$ antiquark.

We note that since the eigenstate method is equivalent to the multiple-scattering model if the inelastic corrections are taken into account, we can obtain all the results in terms of this model. The real part of the hadron-nuclear scattering amplitude is attributable to the inelastic corrections, since the production of a heavier hadron in the intermediate state leads to an additional phase shift of the scattering amplitude.

We shall write the expression for the real part of the hadron-nuclear scattering amplitude for a multichannel problem under the same assumptions that make the Karman-Kondratyuk formula valid

$$\mathrm{Re}\,F_{hA} = -4\pi \int d^2B \int dM^2\,\frac{d^2\sigma_{hN}^{diff}(t=0)}{dM^2\,dt}\,\exp\left[-\frac{1}{2}\sigma_{hN}^{tot}\,T(b)\right]$$
$$\times \int_{-\infty}^{\infty} dl_1 \int_{-\infty}^{\infty} dl_2\,\rho(b, l_1)\,\rho(b, l_2)\sin\left[\Delta q\,(l_2 - l_1)\right]\exp\left[\,i\Delta q\,(l_2 - l_1)\right]. \tag{11}$$

Here $\Delta q = (M^2 - m_h^2)/2E$.

In conclusion, we emphasize the importance of experimental measurement of ReF_{hA}, which is very sensitive to inelastic screening in the nuclei. The determination of ReF_{hA} will make it possible to estimate the parameters of the parton model more accurately.

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