Red spot of Jupiter and the drift soliton in a plasma

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A simplified Rossby wave equation is derived. The solution of this equation is found in the form of a steady-state, isolated vortex that travels along the parallel with a velocity that is higher than the Rossby velocity. The solution has a resemblance to the red spot of Jupiter. The obtained vortex is compared with the drift-convective, isolated vortices in an inhomogeneous plasma.

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As is known, the red spot of Jupiter is an isolated vortex. Such vortices apparently occur because of the twisting of wind by the Coriolis force and retain their shape for a long time. A simplified mathematical description of this effect is of interest.

The following system of equations, which describes the waves in a rotating atmosphere whose depth is much less than the wavelength, was derived in Ref. 1:

$$\frac{d\mathbf{v}}{dt} = -g\nabla h + \Omega[\mathbf{v}\vec{\zeta}]; \quad \frac{d}{dt} = \frac{\partial}{dt} + \mathbf{v}\nabla ; \tag{1}$$

$$\frac{\partial H}{\partial t} + \operatorname{div} H\mathbf{v} = 0; \quad \Omega = 2\omega_0 l; \quad l = \sin\alpha , \tag{2}$$

The atmosphere in this model is represented as an incompressible, shallow fluid with a depth H, $\mathbf{v}$ is the horizontal component of the velocity, g is the gravitational acceleration, ω_0 is the angular rotational velocity of the planet, and ζ is a unit vector in the vertical direction. All quantities depend only on the horizontal coordinates: the meridian angle ϕ and the latitude angle α.

The dispersion equation for small oscillations in this system is

$$\omega[1 + (kr_0/l)^2 - (\omega/\Omega)^2] = -k_\phi v_0/l^2. \tag{3}$$

$$r_0 = (gH_0)^{1/2}/2\omega_0 ; \quad v_0 = gH_0/2\omega_0 R. \tag{4}$$

H_0 is the unperturbed depth, r_0 is the Obukhov length, v_0 is the Rossby drift velocity due to the inhomogeneity Ω, k_ϕ is the projection of the wave vector on the parallel, and R is the planet radius.

Equation (3) describes two branches. The branch, proportional to the right-hand side of Eq. (3), corresponds to Rossby waves. At frequencies much greater than Ω, we have gravity waves in shallow water. We derive a simplified equation that describes only the Rossby waves. We assume that $|\Omega| >> |d/dt|$ consistent with the parameters of the red spot. We obtain from Eq. (1) a series expansion in powers of $1/\Omega$

$$\mathrm{v} = 2\omega_0 r_0^2 [\vec{\zeta} \nabla h]/l - r_0^2 l^{-2} \nabla \partial h/\partial t - 2\omega_0 r_0^4 l^{-3}([\vec{\zeta}\nabla h]\nabla)\nabla h + \dots ;$$

$$h = H/H_0 - 1. \qquad (5)$$

Substituting Eq. (5) in Eq. (2) and omitting comparatively small terms, we obtain the desired, closed equation for h

$$\frac{\partial}{\partial t}(l^2 h - r_0^2 \Delta h) - \frac{v_0}{R}\frac{\partial}{\partial \phi}(h + h^2/2) = 2\omega_0 r_0^4 l^{-1}[\vec{\zeta}\nabla h]\nabla\Delta h ;$$

$$H_0 << L << R \qquad (6)$$

L is the characteristic perturbation length.

We can clearly see from Eqs. (3) and (6) that the Rossby waves are similar to the drift waves in a plasma.[2] The similarity is attributable to the fact that the forces with similar properties are in effect in a rotating atmosphere and in a magnetized plasma: the Coriolis force and the Lorentz force. We must understand that the unperturbed height in Eq. (4) is the height of the homogeneous atmosphere, i. e., $H_0 = v_T^2/g$, where v_T is the thermal velocity of atmospheric particles. Thus, we have from Eq. (4): $r_0 = v_T/2\omega_0$, i. e., the expression for the Obukhov radius is identical to the expression for the Larmor radius of plasma ions.

Equations like Eq. (6) have been obtained in several papers; however, they differ from Eq. (6) in that their terms are of minor importance or their last term in the first part is nonlinear. This nonlinearity is missing in Ref. 2. On the other hand, it is more important than the nonlinearity on the right-hand side of Eq. (6) in the case of Jovian spots or terrestrial anticyclones. In the absence of dispersion, it is responsible for the twisting and flipping of the initial perturbation along the parallel. The nonlinearity on the right-hand side of Eq. (6) causes a more complicated distortion, whose form depends on the characteristic dimensions of the perturbation. The influence of this term on the wave evolution was investigated[2] assuming that it stochasticizes the solution of the equation with time; we, therefore, can use a statistical approach. It was shown that an energy transfer to larger scales occurs at smaller dimensions than Obukhov's dimension (which is called the Rossby radius in the Western literature), i. e., the perturbations are smoothed out. It is possible, however, that the nonlinearities in Eq. (6) cause the formation of structures such as spots and bands.

A steady-state solution in the form of a soliton—an isolated vortex—was obtained[3] for drift waves. It is easy to prove by a direct substitution that Eq. (6) has a similar solution: $h = f(\phi + ut/R, \alpha)$, where f satisfies the equation

$$r_0^2 \Delta f = (I^2 - v_0/u) f - \frac{3}{2} \frac{v_0}{u} f^2 + c\,(f - uI^2/2v_0), \tag{7}$$

where c is an arbitrary function. We must set $c = 0$ for the soliton solution. The constant $-u/R$ is the angular displacement velocity of a soliton along the parallel.

The right-hand side of Eq. (6) contributes to Eq. (7) because of the dependence of I on the latitude. Equation (7) can be easily solved if the soliton dimensions are much smaller than R. Thus, I can be assumed constant in zeroth approximation. At $c = 0$ we obtain the well-known soliton solution, which depends only on r—the distance to the center of a soliton in the horizontal plane. The dependence of I on the latitude can be determined in the next approximation as in the WKB method and then we obtain the approximate solution of (7):

$$f = 1.6(r_0/LI_0)^2 \left(\mathrm{ch}\left\{\frac{3}{4}\frac{r}{L}[1 + \xi(\alpha - \alpha_0)]\right\}\right)^{-4/3}, \tag{8}$$

$$\xi = (L/r_0)^2 \sin 2\alpha_0 ; \quad |\xi| < L/R ; \qquad L > r_0/I_0 .$$

Here I_0 is the value of I at the latitude of the soliton center α_0 and L is the soliton radius. Allowance for the latitude dependence of the coefficient in Eq. (7) gives an oval-shape solution with the vertex directed toward the equator. All the quantities in the soliton are determined by the characteristic radius, which depends on the propagation velocity and the latitude of the soliton center:

$$L = r_0/(I_0^2 - v_0/u)^{1/2}. \tag{9}$$

The value of u is directly proportional to the amplitude and inversely proportional to the size of the vortex. If h is known, we can determine the velocity of matter from Eq. (5), which shows that the rotation of the vortex and the propagation direction are opposite to the planet rotation. Like an anticyclone, the pressure in the middle of the vortex is higher than at the edges.

Acceptable data for the Jovian spots, which are in qualitative agreement with the obtained solution, were recently obtained.[4] The Jovian parameters are: $R = 7.1 \times 10^7$ m, $\omega_0 = 3.5 \times 10^{-4}$ rad/sec, $r_0 = 2 \times 10^6$ m, and $v_0 = 25$ m/sec. We have $r_0 = 900$ km and $v_0 = 40$ m/sec for the earth. These values can be effectively smaller when a layer of cold air is near the surface or for other reasons. If this fact and the possibility of wind drift are taken into account, then the anticyclones on the earth will resemble the solution (8). The amplitude of the soliton (8) is determined from the equilibrium between the dissipation and the energy pumping, which are ignored here and which are attributable to a weak, large-scale instability in the atmosphere. These vortices can be easily produced in the laboratory in a shallow liquid in a rotating container with a parabolic shape of the bottom.

The solution (8) shows that the existence of solitons in the atmosphere does not require the presence of an H_0 gradient, as assumed in Ref. 5.

The main difference between the vortex (8) and the drift solitons in a plasma[3] is that the main gradient in the Rossby waves is the Coriolis force and the main gradients in the drift solitons are the temperature and density gradients of the plasma.

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