Enhanced transmission of an unoriented mesophase of nematics for the ordinary wave

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It is shown that an ordinary wave, as it propagates through a nonuniformly oriented liquid crystal, has a nearly constant phase velocity and therefore it is scattered weakly. The results are important for studying the disclinations in liquid crystals by using optical methods.

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As is known, the mesophase of a nematic liquid crystal (NLC) is usually a turbid liquid. The reason for this is that the orientation of the director in large volumes is highly nonuniform. The refractive index of the extraordinary (e) wave is strongly dependent on the angle θ between the director vector **d** and the propagation direction $\mathbf{k}/|\mathbf{k}|$;

$$n_e(\theta) = \left[\frac{\cos^2\theta}{n_\perp^2} + \frac{\sin^2\theta}{n_\parallel^2}\right]^{-1/2} \approx n_\parallel + (n_\perp - n_\parallel)\cos^2\theta(\mathbf{r}). \qquad (1)$$

Since $n_\parallel - n_\perp \sim 0.1 - 0.3$ for a typical NLC, the fluctuations of the refractive index

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$n_e(\mathbf{r})$ are large. The deviation of a ray in geometrical-optics approximation with a transverse gradient $d\theta/dr \sim a^{-1}$ within a length l amounts to $\alpha \sim l(n_\perp - n_\parallel)/a$; even for one inhomogeneity with $l \sim a$ this gives $\alpha \sim 0.1$–0.3. Therefore, the passage of an e wave through several inhomogeneities completely randomizes the propagation direction, which accounts for the turbid medium.

We focus attention in this paper on the simple fact that the variations in the direction of the director for an ordinary (o) wave do not change the refractive index; $n_o = n_\perp$ and do not depend on θ. Therefore, in the crudest approximation the o wave propagates as though it were in an optically homogeneous medium, i.e., not a turbid medium.[1] A more exact analysis of the problem shows that the o wave is distorted as it propagates through an NLC with nonuniform orientation of the director, but the distortion is markedly smaller than for the e wave.

First, we calculate the correction for the local, effective, refractive index—the correction due to orientational nonuniformity. We look for the induction vector $\mathbf{D}(\mathbf{r})$ in the o light wave in the form

$$\mathbf{D}(\mathbf{r}) = e^{ik_o z}\{\mathbf{e}_o(\mathbf{r})A(\mathbf{r}) + \mathbf{e}_e(\mathbf{r})B(\mathbf{r}) + \mathbf{e}_z(\mathbf{r})C(\mathbf{r})\}. \tag{2}$$

Here $\mathbf{k}_0 = \omega n_o/c \equiv \omega n_\perp/c$, and the unit vectors $\mathbf{e}_o(\mathbf{r})$ and $\mathbf{e}_e(\mathbf{r})$ pertain to the o and e polarizations, which follow exactly the local orientation of the optical axis

$$\mathbf{e}_o(r) = [\mathbf{d}(\mathbf{r}) \times \mathbf{e}_z]/|[\mathbf{d}(\mathbf{r}) \times \mathbf{e}_z]|; \quad \mathbf{e}_z(r) = [\mathbf{e}_o(r) \times \mathbf{e}_z]. \tag{3}$$

It can be shown that $B \sim A\lambda|\text{grad}\,\mathbf{d}|/(n_e - n_o)$ and $C \sim A\lambda|\text{grad}\,\mathbf{d}|$ for the o wave. Assuming that $n_e - n_o \ll n_o$, we drop for this reason the terms that are $\sim C\mathbf{e}_z$ in Eq. (2). The Maxwell's equations have the form

$$\text{rot rot}\,\mathbf{E} - \omega^2\mathbf{D}/c^2 = 0, \tag{4}$$

$$\mathbf{E} = \hat{\epsilon}^{-1}\mathbf{D}; \quad \epsilon_{ik}(\mathbf{r}) = n_\perp^2\delta_{ik} + (n_\parallel^2 - n_\perp^2)d_i(\mathbf{r})d_k(\mathbf{r}).$$

Writing the director unit vector $\mathbf{d}(\mathbf{r})$ in the form

$$\mathbf{d}(\mathbf{r}) = (\mathbf{e}_x\cos\phi(\mathbf{r}) + \mathbf{e}_y\sin\phi(\mathbf{r}))\sin\theta(\mathbf{r}) + \mathbf{e}_z\cos\theta(\mathbf{r}) \tag{5}$$

from Eq. (4) in the approximation of slowly varying amplitudes $A(\mathbf{r})$ and $B(\mathbf{r})$ for $|B| \ll |A|$, i.e., for the o wave, we obtain the following equations by projecting Eq. (4) to $\mathbf{e}_o$ and $\mathbf{e}_e$:

$$\frac{\partial A}{\partial z} - \frac{n_\perp^2}{n_e^2(\theta)}\frac{\partial\phi}{\partial z}B = 0, \tag{6a}$$

$$B = -i\frac{2n_\perp c}{\omega(n_e^2 - n_\perp^2)}\frac{n_e^2}{n_\perp^2}\frac{\partial\phi}{\partial z}A. \tag{6b}$$

The relation (6b) means that because of the imprecise uniformity of the director $\partial\phi/\partial z$ an e wave is locally added to the o wave at each point, which makes the resulting polarization slightly elliptical (compare it with the result for slightly twisted cholesterics,[2] p. 264). The relative amplitude of the addition is of the order of $|B/A| \sim l_{syn}\,\partial\phi/\partial z$, i.e., it is determined by the rotation angle of the director azimuth $\Delta\phi$ within the length $l_{syn} = \lambda/(n_e - n_o)$ of the phase mis-synchronization of the e and o waves.

Substitution of Eq. (6b) in Eq. (6a) gives for $|n_e - n_o| \ll n_o$

$$\frac{\partial A}{\partial z} = i\frac{\omega}{c}\delta n_{\text{eff}}(\mathbf{r})A; \quad \delta n_{\text{eff}} = -\left(\frac{c}{\omega}\right)^2[n_e(\theta(\mathbf{r})) - n_o]^{-1}\left(\frac{\partial\phi}{\partial z}\right)^2. \tag{7}$$

Under typical conditions the phase fluctuations of the transmitted wave due to δn_{eff} from Eq. (7) are small. In fact, suppose that the characteristic size of the inhomogeneity is $a \sim 10^{-2}$ cm, $|\partial\phi/\partial z| \sim 10^2$ cm^{-1}, $\lambda_{vac} = 6 \times 10^{-5}$ cm, the total cell thickness is $L = 0.5$ cm, and $n_e - n_o \sim 0.5(n_{\parallel} - n_{\perp}) \sim 0.1$. Thus, $\int \delta k\,dz \sim 0.5$, i.e., the phase is equal to 0.5 radian, on the average. Its transverse fluctuations are approximately $\sqrt{L/a} \approx 7$ times smaller and therefore are negligibly small.

In fact, the main contribution to the distortion of the wave passing through the cell is the spatial dependence of the unit vector $\mathbf{e}_o(\mathbf{r})$, which is the coefficient of the amplitude $A(\mathbf{r})$ in Eq. (2). If the director is randomly oriented at the cell walls, then the projection of $A(\mathbf{r}_\perp, z=0) = \mathbf{E}_{inc}\mathbf{e}_o(\mathbf{r}_\perp, z=0)$ on the o wave will have severe transverse inhomogeneities at the entrance to the cell; this also applies to the inhomogeneity of the unit vector $\mathbf{e}_o(\mathbf{r}_\perp, z=L)$ at the exit. Thus, the scattering angle of the transmitted o wave is $\Delta\theta \sim \lambda/a_\perp$, where $a_\perp$ is the transverse inhomogeneity of the unit vector.

If the director has a uniform orientation (planar or homeotropic[2]) at the cell walls because of their rubbing, then the change of the unit vector $\mathbf{e}_o$ from the entrance to the exit will correspond to the change of its sign. The presence or absence of such a sign change for the unit vector $\mathbf{e}_o$, which follows the director adiabatically, is determined only by the topological properties of the director distribution in the volume along the path of the rectilinear ray of the o wave. In other words, the unit vector changes its sign as a result of transverse shift of the ray that intersects a disclination of the appropriate type. A more detailed investigation of this problem lies outside the scope of this paper. We shall also forego a detailed examination of the region of applicability of Eq. (7); it is clear that this equation is invalid as $n_e - n_o \to 0$, i. e., for the propagation along the director.

Thus, we predict that a relatively thick cell with orienting walls and a nematic liquid crystal that is poorly oriented in its volume scatter the incident, plane o wave at comparatively small angles $\Delta\theta \sim \lambda/a$, where a is the characteristic transverse distance between the projections of the disclination lines on the cell walls.

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[1)] After the completion of this work, we learned that such a qualitative confirmation was given independently by S. M. Arakelyan, L. E. Arushanyan, and Yu. S. Chilingaryan.[1]

[2]The wave must be obliquely incident on the cell in order to excite a pure *o* wave in the homeotropic case.

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