The possibility of low-frequency, electromagnetic waves in metals

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It is shown that low frequency, electromagnetic waves with a frequency lower than that of the plasma frequency can propagate at low temperatures in metals in which the electrical conductivity and thermoelectric current are produced by electrons of several bands, if the metal has a temperature gradient. We call these waves thermoelectromagnetic waves.

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It was shown in Ref. 1 that if a metal has a strong magnetic field, then the electromagnetic wave incident on it can pass through the metal, even if its frequency is lower than the plasma frequency ω_p.

This paper is devoted to another case in which weakly damped electromagnetic waves with a frequency $\omega < \omega_p$ can occur in a metal. These are called thermoelectromagnetic waves, i.e., electromagnetic waves occurring in the presence of a temperature gradient $\vec{\nabla} T$.[2,3] We show that at sufficiently low temperatures $\vec{\nabla} T$ gives rise to the propagation of weakly damped waves with a frequency $\omega < \omega_p$ in metals in which the electrical conductivity and thermoelectric current are produced by electrons in several energy bands. Such waves were recently detected in bismuth by Kopylov.[4,5]

1. Thermoelectric field produced as a result of increasing the number of electrons in several bands by phonons. The kinetic equation in the collision-frequency ν approximation has the form

$$\frac{\partial \epsilon}{\partial \mathbf{p}} \frac{\partial f_p}{\partial \mathbf{r}} - \frac{\partial \epsilon}{\partial \mathbf{r}} \frac{\partial f_p}{\partial \mathbf{p}} + \nu (f_p - f_p^\circ) = 0,$$

where $f_p(d\mathbf{p})$ is the distribution function, which is normalized to the concentration n of electrons, $f_p^o = f_p^o(\varepsilon - \mu)/T = f_p^o(\xi)$ is the equilibrium distribution function, μ is the chemical potential, T is the temperature in energy units, and $\varepsilon(p)$ is the electron energy such that $\partial\varepsilon/\partial\mathbf{p} = \mathbf{v}$ (velocity). If ∇T is present and if the "phonon wind" is taken into account, then the equation in the linear approximation will have the form

$$-\mathbf{v} \frac{\vec{\nabla} T}{T} \frac{\partial f_p^\circ}{\partial \xi} \left(\xi + \frac{\partial \mu}{\partial T} + \chi \right) - e\mathbf{E}_\circ \frac{\mathbf{v}}{T} \frac{\partial f_p^\circ}{\partial \xi} + \nu f_1(p) = 0, \qquad (1)$$

where the term $(-\chi \vec{\nabla} T)$ describes a force produced by the phonon wind, $\chi(\varepsilon)$ is a dimensionless value that characterizes the drag (mutual drag), and E_o is the thermo-

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electric field. If the values of the order of T/μ are disregarded, then $e\mathbf{E}_o = -\alpha \vec{\nabla} T$, where $\alpha = \chi(\mu)$. $\chi(\mu) \approx 1$ for metals. If an electromagnetic wave is propagated in the metal, then the following equation must be solved in order to determine the alternating current proportional to ∇T:

$$\left(\frac{\partial}{\partial t} = \mathbf{v}\vec{\nabla}_{\mathbf{r}} + \nu\right) f_p^1(t) = \frac{e}{mc}[\mathbf{v}\,\mathbf{B}]\frac{\partial f_1(p)}{\partial \mathbf{v}}, \tag{2}$$

where B is the magnetic field of the wave. We shall show that $\omega << \nu$ for thermoelectromagnetic waves and $\mathbf{k}\cdot\mathbf{v} < \nu$ for a number of metals. We shall, therefore, disregard ω and, for simplicity, also $\mathbf{k}\cdot\mathbf{v}$. Hence, the alternating current $\mathbf{j} \sim [\mathbf{B}\vec{\nabla} T]$ is of the order of $(T/\mu)^2$; if T/μ is disregarded, then $j = 0$. The situation changes if the electrons in several energy bands are taken into account in the electrical conductivity and in the thermoelectric current. For simplicity, we analyze two bands a and b. In this case,

$$f_1(p) = \frac{\mathbf{v}}{T}\left[eE_o\left(\frac{1}{\nu_a}\frac{\partial f^o_{pa}}{\partial \xi_a} + \frac{1}{\nu_b}\frac{\partial f^o_{pb}}{\partial \xi_b}\right) + \vec{\nabla} T\left(\frac{\chi_a}{\nu_a}\frac{\partial f^o_{pa}}{\partial \xi_a} + \frac{\chi_b}{\nu_b}\frac{\partial f^o_{pb}}{\partial \xi_b}\right)\right] \tag{3}$$

hence, $e\mathbf{E}_o = -\alpha_{ab}\vec{\nabla} T$.

$$a_{ab} = \frac{\chi_a n_a m_b \nu_b + \chi_b n_b m_a \nu_a}{n_a m_b \nu_b + n_b m_a \nu_a}. \tag{4}$$

Substituting Eq. (3) in the equation for f'_a and f'_b functions, which are analogous to Eq. (2), we can see that one part of the current j that is proportional to ∇T has to form

$$\mathbf{j} = \eta[\mathbf{B}\vec{\nabla} T], \quad \eta = C_{ab}(\chi_a - \chi_b)(m_a\nu_a - m_b\nu_b),$$

$$C_{ab} = \frac{e^2}{c}\frac{1}{m_a m_b \nu_a \nu_b}\frac{n_a n_b}{n_a m_b \nu_b + n_b m_a \nu_a}. \tag{5}$$

This current generally does not have a small T/μ. The Nernst-Ettingshausen coefficient η can be both positive and negative

2. Frequencies, wave vectors, and phase velocities of weakly damped, thermoelectromagnetic waves. Substituting the total current

$$\mathbf{j} = \sigma\mathbf{E} + \eta[\mathbf{B}\vec{\nabla} T], \quad \sigma = \sigma_a + \sigma_b$$

in the Maxwell equation, we obtain the equation

$$[\kappa\omega^2 - c^2k^2 + 4\pi i\omega\sigma + 4\pi i c\eta(\mathbf{k}\vec{\nabla}T)]\mathbf{E} = -4\pi i c\eta\mathbf{k}(\mathbf{E}\nabla T), \quad (6)$$

where κ is the dielectric constant of the lattice. If $(E\vec{\nabla}T)\neq 0$, then the waves will be longitudinal. If, however, $(E\vec{\nabla}T)=0$, then they will be transverse waves and the dispersion relation for them will have the form

$$c^2k^2 - 4\pi ic\eta(\mathbf{k}\vec{\nabla}T) - 4\pi i\omega\sigma - \kappa\omega^2 = 0. \quad (7)$$

Hence, k is

$$k = k' + ik'' = \frac{2\pi i\eta\nabla T}{c} \pm \frac{1}{c}[4\pi i\omega\sigma - (2\pi\eta\nabla T)^2]^{1/2}. \quad (8)$$

The waves will be weakly damped if $k''<<k'$; a solution with a minus sign in front of the radical and the inequality

$$\omega << \omega_{max} = \pi(\eta\nabla T)^2/\sigma \quad (9)$$

must be used in this case. If $\omega=\gamma\omega_{max}$, $\gamma<<1$, then $k''/k'=\gamma<<1$, so that the waves are weakly damped. For these waves,

$$k' = -\frac{\omega\sigma}{c\eta\nabla T} = \gamma k'_{max},$$
$$k'_{max} = -\frac{\omega_{max}\sigma}{c\eta\nabla T} = -\frac{\pi\eta\nabla T}{c}. \quad (10)$$

Therefore, the weakly damped waves are propagated in the direction $(-\vec{\nabla}T)$ when $\eta>0$ and in the direction $(+\vec{\nabla}T)$ when $\eta<0$. Their phase velocity is

$$u = \omega/k = c\eta\nabla T/\sigma, \quad (11)$$

i.e., it is independent of the frequency. The other solution, which corresponds to the plus sign in front of the radical in Eq. (8), gives strongly damped waves that propagate in the direction $(\pm\vec{\nabla}T)$ when $\eta\gtrless 0$. For them $k''/k'=1/\gamma>>1$. The estimates in Sec. 4 show that the relation ω/ν ignored by us is $<<\omega/\omega_{max}$; we used this relation to expand in a series the expression for k, so that the expansion is valid.

3. Influence of spatial dispersion. The σ and η coefficients receive the same additional multiplier if the spatial dispersion is taken into account

$$-\frac{3}{4}\left[\frac{2}{a^2} + \frac{i}{a}\left(1+\frac{1}{a^2}\right)\ln\frac{1+ia}{1-ia}\right], \quad (12)$$

where $a=kv/\nu=kl$; l is the length of the free path of electrons, which for simplicity, we assume to be the same in both bands. At $a<<1$ the expression (12) reduces to the multiplier $(1-\frac{1}{2}k^2l^2)$. Substituting it in Eq. (7) and replacing k by $k+\delta k$, we find that

$$
\delta k' = \frac{k}{32}\left(\frac{\omega}{\sigma}\right)^2\left(\frac{c}{u}\right)^4\left(\frac{l\omega}{u}\right)^2,
$$
$$
\delta k'' = -\frac{k}{16}\,\frac{\omega}{\sigma}\left(\frac{c}{u}\right)^2\left(\frac{l\omega}{u}\right)^2, \tag{13}
$$

for weakly damped waves when $\eta > 0$. The input values are everywhere assumed to be absolute in these expressions. At $\eta < 0$ the signs in front of $\delta k'$ and $\delta k''$ are reversed. Since, according to Sec. 2, the waves in which $k' < 0$ [i.e., the waves that propagate in the direction $(-\vec{\nabla} T)$] are weakly damped when $\eta > 0$, and the waves with $k' > 0$ are weakly damped when $\eta < 0$, we can see from the given expressions (13) that the ratio k'/ω decreases with increasing frequency in both cases, i.e., the phase velocity of the waves increases. The damping of waves also increases in both cases, although slower than the phase velocity. The waves are strongly damped in the opposite, limiting case $a \gg 1$.

4. Estimates and comparison with experiment at $T = 4.2$ K (χ grad $T \approx$ 4 grad $\cdot$ cm^{-1}). Since $\nu \approx 4 \times 10^{10}$ for Cu,[6] Eqs. (9), (10), and (11) give the values $\omega_{max} \approx 6 \times 10^2$, $k_{max} \approx 10^2$, and $u \approx 6$.

Cd : $\nu \approx 10^{10}$, $\omega_{max} \approx 10^4$, $k_{max} \approx 3 \times 10^2$, $u \approx 30$.

Al : $\nu \approx 2.4 \times 10^9$ [7], $\omega_{max} \approx 1.6 \times 10^5$, $k_{max} \approx 1.2 \times 10^3$, $u \approx 1.3 \times 10^3$.

Mo : $\nu \approx 10^9$ [8], $\omega_{max} \approx 2 \times 10^6$, $k_{max} \approx 4 \times 10^3$, $u \approx 5 \times 10^2$.

Bi : $\nu \approx 10^7 - 10^9$ [9], $\omega_{max} \approx 5 \cdot 10^4 - 5 \cdot 10^7$, $k_{max} \approx 20 - 10^3$, $u \approx 10^3 - 10^5$

At $\omega/\omega_{max} = k/k_{max} \approx 0.1$ these waves are relatively weakly damped. The spatial dispersion can lead to a noticeable deviation from the relation $k \sim \omega$ for Al, Mo, and Bi.

The theory discussed by us is in good agreement with the results of the experiments of Kopylov[3,4] for bismuth. His results also lead to a one-directional propagation of the thermoelectromagnetic waves, to a weak frequency dependence of their phase velocity at low frequencies, and to an increase of the phase velocity at higher frequencies. The wavelengths in bismuth determined by us also agree with the experimental data to within an order of magnitude.

Our theory makes it possible to estimate the drag constant χ from the experimental data for the Nernst-Ettinghausen coefficient η.

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