Electrodimensional resonance in semiconductor films

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The presence of a static-current resonance in semiconductor films is theoretically predicted when the distances between the Stark and the film sublevels coincide.

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We investigate a semiconductor film placed in a constant electric field **E** that lies in its plane. We start with the Hamiltonian that describes the interaction of electrons with phonons in the situation under investigation

$$\hat{H} = \sum_{\mathbf{p},s} \epsilon_s (\mathbf{p} + e\mathbf{E}t) a^+_{\mathbf{p}s} a_{\mathbf{p}s} + \sum_{\mathbf{q}} \omega_{\mathbf{q}} b^+_{\mathbf{q}} B_{\mathbf{q}}$$

$$+ \sum_{\mathbf{p},\mathbf{q}} \sum_{s,s'} C^{ss'}_{\mathbf{q}} a^+_{\mathbf{p}s} a_{\mathbf{p}-\mathbf{q}s'} (B_{\mathbf{q}} + B^+_{-\mathbf{q}}).$$

Here $\epsilon_s(\mathbf{p}) = \epsilon_s + \epsilon(\mathbf{p})$ is the electron dispersion law, ϵ_s are the single-particle levels in the potential well that modulates the film, $\mathbf{p} = \{p_{||}, p_\perp\}$ is a two-dimensional momentum in the plane of the film, $p_{||}$ and $_\perp$ are the components in the direction of the electric field **E** and at right angles to it, $a^+_{\mathbf{p}s}, a_{\mathbf{p}s}$ $(B^+_{\mathbf{q}}, B_{\mathbf{q}})$ are the electron (or phonon) production and annihilation operators, $\omega_{\mathbf{q}}$ is the frequency of a phonon with the wave vector **q**, and $C^{ss'}_{\mathbf{q}}$ are the matrix elements of electron-phonon interaction; the system of units in which $\hbar = 1$ is used. In writing the Hamiltonian the elastic constant of the film was assumed to be close to that of the substrate, so that the phonons can be assumed to be nonquantized.

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We shall follow the method developed in Ref. 1 in the derivation of the quantum kinetic equation. Assuming that the phonons are in equilibrium and that the electron gas is nondegenerate, we obtain in the approximation of the weak electron-phonon coupling the equation for the distribution function of electrons due to the canonical momentum

$$\frac{\partial \phi_s(\mathbf{p}, t)}{\partial t} = \{2\sum_{\mathbf{q},s'} |C_{\mathbf{q}}^{ss'}|^2 \mathrm{Re} \int_{-\infty}^{t} dt' \exp\left(i \int_{t'}^{t} dr \left[\epsilon_s(\mathbf{p} + e\mathbf{E}r) - \epsilon_{s'}(\mathbf{p} - \mathbf{q} + e\mathbf{E}r) - \omega_{\mathbf{q}}\right]\right) \left[\phi_{s'}(\mathbf{p} - \mathbf{q}, t') N_q - \phi_s(\mathbf{p}, t')(N_q + 1)\right]\} - \{\mathbf{p} \to \mathbf{p} + \mathbf{q}, s \leftrightarrow s'\}, \quad (1)$$

where $\{\mathbf{p} \to \mathbf{p} + \mathbf{q}, s \leftrightarrow s'\}$ represents the explicitly written expression inside the braces with the appropriate replacement of arguments and permutation of the indices and N_q is the phonon distribution function. In contrast to the kinetic equation of Ref. 2, Eq. (1) makes it possible to investigate the region of strong constant fields in which the Stark quantization of the electron motion is appreciable. The absence of terms in Eq. (1) that describe the direct, band-to-band transitions due to the action of the electric field[2] is attributable to the direction of the field **E** (in the plane of the film) and to the dispersion law (the motion in the direction of the film is separated from that at right angles to it).

We assume that the constant field is sufficiently strong, so that $\Omega\tau \gg 1$, where $\Omega = 2\pi eE/G_{||}$ is the Stark frequency, $\mathbf{G}_{||}$ is the smallest vector of the reciprocal-lattice vectors, which coincides with the direction of **E** and τ is the relaxation time. Since $\epsilon(\mathbf{p})$ is a periodic function of $p_{||}$ with the reciprocal-lattice period, we have

$$\epsilon(\mathbf{p}) = \sum_k \tilde{\epsilon}_k(p_\perp) \exp\left[i 2\pi k p_{||}/G_{||}\right]. \quad (2)$$

Substituting Eq. (2) in Eq. (1) and solving Eq. (1) by iterations[3] over $(\Omega\tau)^{-1} \ll 1$ with allowance for the standard expression for the current

$$j_{||} = e \sum_{\mathbf{p},s} \frac{\partial \epsilon_s(\mathbf{p} + e\mathbf{E}t)}{\partial p_{||}} \phi_s(\mathbf{p}, t)$$

in the case of elastic scattering by acoustic phonons ($\omega_{\mathbf{q}} \ll T, \Delta_\perp$, $N_q \gg 1$, and $\Delta_\perp$ is the width of the conduction band at right angles to the constant field) we obtain after integration with respect to $p_\perp$ and $q_\perp$ and using the new variables $\tilde{\epsilon}_0(p_\perp) = \epsilon$ and $\tilde{\epsilon}_0(q_\perp) = \epsilon'$,

$$j_{||} = e \sum_{s,s'} \sum_{n \neq 0} \int d\epsilon d\epsilon' G_n^{ss'}(\epsilon, \epsilon') g(\epsilon) g(\epsilon') \delta\left[\epsilon' + \epsilon_{s'} - \epsilon_s + n\Omega - \epsilon\right] \left[\phi_{s'}(\epsilon') - \phi_s(\epsilon)\right], \quad (3)$$

where $g(\epsilon)=dp_\perp/d\epsilon$ is the density of states and $G_n^{ss'}(\epsilon,\epsilon')$ is a smooth function of the energy and of the quantum numbers which are generally expressed in terms of infinite products of Bessel functions. $\phi_s(\epsilon)$ is the electron distribution function[3,4] in zeroth order with respect to $(\Omega\tau)^{-1}\ll 1$.[3,4]

Since each density of states in Eq. (3) has integrable singularities of the type

$$g(\epsilon)\sim[(\Delta_\perp/2)^2-\epsilon^2]^{-1/2}$$

near the bottom and the top of the corresponding subband, $j_{||}$ is a smooth function of the electric field E, except for those E values at which $n\Omega$ is close to $\epsilon_s-\epsilon_{s'}$, $\epsilon_s-\epsilon_{s'}+\Delta_\perp$, or $\epsilon_s-\epsilon_{s'}-\Delta_\perp$. After dropping in Eq. (3) the regular terms $j_{||}^{reg}$ and the regular cofactors in the resonance terms $j_{||}^{res}$ and integrating with respect to ϵ and ϵ', we obtain $j^{res}=\Sigma_{ss'}j_{ss'}$, where $j_{ss'}\sim\mathbf{K}(\sqrt{1-\delta^2})\,[1-\delta^2]$. Here $\mathbf{K}(k)$ is a complete, first-order elliptic integral with the modulus k, $\theta\,[x]=0$ for $x<0$ and $\theta\,[x]=1$ for $x\geq 0$, and $\delta=(n\Omega+\epsilon_{s'}-\epsilon_s)/\Delta_\perp$. Thus, $n=1,2\ldots$ $j_{||}^{res}$ has logarithmic singularities at $n\Omega=\epsilon_s-\epsilon_{s'}$ and it has discontinuities of finite magnitude at $n\Omega=\epsilon_s-\epsilon_{s'}\pm\Delta_\perp$, $n=1,2\ldots$.

By repeating the arguments presented above, we can easily obtain similar results for the inelastic scattering by optical phonons ($\omega_q=\omega_0\gg T, N_q\ll 1$, and T is the temperature in energy units). In this case $j_{||}^{res}$ has logarithmic singularities at $n\Omega=\epsilon_s-\epsilon_{s'}+\omega_0$, $n=1,2\ldots$ and has discontinuities of finite magnitude at $n\Omega=\epsilon_s-\epsilon_{s'}+\omega_0\pm\Delta_\perp$, $n=1,2\ldots$. At $s=s'$ the obtained conditions are identical to the known conditions of electrophonon resonance,[4] which were obtained on the assumption that the density of states in a two-dimensional band has no singularities.

Since the current density is $j_{||}=j_{||}^{reg}+j_{||}^{res}$, the effects described most clearly occur at $j_{||}^{res}>j_{||}^{reg}$. Such a situation is realized in a quantizing electric field when $\Omega\sim\Delta_\perp$. If the width Δ_g of the forbidden band exceeds the width Δ of the conduction band, then the probability of band-to-band breakdown will be small for the parameter $\exp[-\gamma\Delta_g^{3/2}/\Omega\Delta^{1/2}]$ ($\gamma\sim 1$) up to very strong fields $\Omega\sim\Delta_g(\Delta_g/\Delta)^{1/2}>\Delta$. Organic semiconductors like anthracene, where $\Delta_g\sim 1$ eV$\gg\Delta\sim 0.025$ eV, have the required parameters.[5]

In conclusion, we note that the described effects in a strong, constant electric field will appear in all physical quantities that are expressed in terms of the density of states (i.e., quantities such as the electromagnetic wave and sound absorption coefficients, etc.).

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