# Quark-lepton families: from $SU(5)$ to $SU(8)$ symmetry[1)]

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A unified model of weak, electromagnetic, and strong interactions of constituent quarks and leptons with a local $SU(8)$ symmetry is proposed.

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The Giorgi-Glashow $SU(5)$ model, which unifies in a minimal way the Weinberg-Salam theory with quantum chromodynamics (QCD), gives an isolated description of each of the three, currently existing quark-lepton families $(u, d, \nu_e, e)$, $(c, s, \nu_\mu, \mu)$, and $(t?, b, \nu_\tau, \tau)$.[1)] Because of this, both the complete systematics of quarks and leptons and the effects associated with Cabibbo quark mixing, with *CP* violation, etc. remain outside the scope of $SU(5)$ unification.

It seems to us that the two groups of fundamental quantum numbers–color $i_c = 1, 2, 3$ $[SU(3)_c]$ and flavor $i_f = 1, 2$ $[SU(2) \otimes U(1)]_f$–must be supplemented by an analogous group of "weighting," non-abelian quantum numbers $i_h = 1, 2, 3$ $[SU(3)_h]$ that number the families. It can be assumed, therefore, that these eight quantum numbers constitute the fundamental representation of the $SU(8)$ group and the quarks and leptons "are seated" in one of its (irreducible or reducible) representations.

We can go further, however, and connect this fundamental representation with the fields of subelementary fermions-preons[2] $\psi_\alpha(\alpha = 1...8)$, which assemble into quarks and leptons as a result of additional interaction between themselves, analogous to that in QCD, with a certain unbroken symmetry group $M$. The confinement radius of this group is very small $[z^{-1} \geqslant \mu = 0\,(10^{15}\ \mathrm{GeV}]^{3}$ and the ordinary quarks and leptons comprised of preons are colorless with respect to the $M$ group, just as the hadrons are colorless with respect to the $SU(3)_c$ group.

A simple example of the "metacolor" group $M$, which corresponds to a 3-preon configuration for the constituent quarks and leptons and which eliminates condensation of preon pairs $\langle \bar{\psi}_L \psi_R \rangle \neq 0$ (this would apparently indicate the existence of masses $\sim\mu$ in the observed quark-lepton spectrum[4]), has the form

$$M(i_L, m_R) = SO(3)_L \otimes SO(3)_R, \quad \psi_{\alpha i, L}\,(= 1, 2, 3), \quad \psi_{\alpha m, R}(m = 1, 2, 3). \tag{1}$$

We shall "collect" the quarks, leptons and scalar mesons from the fields of Eq. (1). It is clear that the unique, $S$-wave, metacolor singlets with spin ½ form the configurations

  

$$\psi_{\alpha i,L}\psi_{\beta j,L}\tilde{\psi}^{\gamma}_{kL}\epsilon^{ijk}\ \psi^{\gamma}_{[\alpha\beta],L}(216), \qquad \psi_{\alpha i,L}g^{(L)}_{i}\sim\psi_{\alpha,R}\ (8), \tag{2}$$

where $g^{(L)}_i$ denotes a gluon of the $SO(3)_L$ group and $\tilde{\psi}^{\gamma}_{i,L}$ is a charge conjugation above the field $\psi_{\gamma i,L}$: $\tilde{\psi}^{\gamma}_{i,L}=C(\overline{\psi_{\gamma i,L}})^T$. The dimensions of the corresponding $SU(8)$ and $SU(5)\otimes SU(3)$ group representations are indicated in the parentheses—whose expansion has the form

$$216 = (\overline{45},1) + (24,3) + (5,8) + (1,6) + (5,1) + (1,3) + (\overline{5},\overline{3}) + (10,\overline{3}),$$
$$8 = (5,1) + (1,3). \tag{3}$$

Such representations of $\tilde{\psi}^{\gamma}_{[\alpha,\beta],R}$ (216) and $\psi_{\alpha L}$ (8) form right-handed preon fields $\psi_{\alpha m,R}$ and gluons of the $SO(3)$ group. The observed quark-lepton families should evidently be identified with the fragment $(\overline{5},\overline{3})_L+(10,\overline{3})_L$ of the $216_L$ representation. The remaining fermions in the given representation $8_R+216_L+8_L+216_R$ receive large masses through vacuum expectations of the constituent scalar mesons.

The scalar particles originate from the compositions

$$\tilde{\psi}^{\alpha}_{i,L}\psi_{\beta i,L}\sim\phi^{(1)\alpha}_{\beta}\ (63), \qquad \tilde{\psi}^{\alpha}_{m,R}\psi_{\beta m,R}\sim\phi^{(2)\alpha}_{\beta}(63),$$

$$\psi_{\alpha i,L}\psi_{\beta j,L}g^{(L)}_{k}\epsilon^{ijk}\sim\phi^{(1)}_{[\alpha\beta]}(28), \quad \psi_{\alpha m,R}\psi_{\beta n,R}g^{(R)}_{p}\epsilon^{mnp}\phi^{(2)}_{[\alpha\beta]}(28) \tag{4}$$

with $SU(5)\otimes SU(3)$ expansion

$$63 = (1,1) + (24,1) + (1,8) + (\overline{5},3) + 5,\overline{3}), \qquad 28 = (1,\overline{3}) + (5,3) + (10,1). \tag{5}$$

In addition to the main fields (4), which presumably generate large ($\sim\mu$) vacuum expectations for the (1, 1), (24, 1), and (1, 3) components, there can also exist multipreon (4, 6, ...), metacolor-singlet compositions (exotic preons)

$$\phi^{(1,2)}_{\{\alpha\beta\}}(36), \qquad \phi^{(1,2)\alpha}_{[\beta\gamma\delta]}(420), \qquad \phi^{(1,2)}_{[\gamma\delta\epsilon\eta]}(2072) \tag{6}$$

whose vacuum expectations for the (5, 3), (5, 6), and (45, 3) components are small, $\mu_1\sim 0\,(10^2$ GeV). Thus, the hierarchy of vacuum expectations can have a dynamic origin. The breakdown scheme of the original $SU(8)$ symmetry has the form

$$SU(8)\xrightarrow{63^{(1,2)}}SU(3)_c\otimes[SU(2)\otimes U(1)]_f\otimes[SU(3)\otimes U(1)]_L\longrightarrow$$
$$\xrightarrow{28^{(1,2)}}SU(3)_c\otimes[SU(2)\otimes U(1)]_f\xrightarrow{36^{(1,2)},\ldots}SU(3)_c\otimes U(1)_{EM} \tag{7}$$

We must strongly emphasize that the $SU(5)\otimes SU(3)$ decay channel of $SU(8)$ is dominant only if the fields $\phi_{\beta}^{(1,2)\alpha}$ are present in the Higgs polynomial, in addition to the fourth-power, self-interaction terms as well as a noticeable cubic term. If this term is missing or is small, then the $SU(8) \to SU(4) \otimes SU(4) \otimes U(1)$ channel will dominate and the intermediate $SU(5)$ systematics will be missing.

Let us discuss in greater detail the mass spectrum of fermions. The main fields in Eq. (4) do not participate in the formation of masses of the observed quarks and leptons; this is accomplished by multipreon, exotic scalars

$$L_Y^{(1)} = G_1^{(1)} \bar{\psi}^{\alpha} \psi^{\gamma}_{[\alpha\beta]} \phi_{\gamma}^{(1)\beta} + G_2^{(1)} \psi^{\alpha}_{[\beta\delta]} C \psi^{\delta}_{[\alpha\eta]} \phi^{(1)\{\beta\eta} + G_3^{(1)} \psi^{\alpha}_{[\beta\delta]} C \psi^{\delta}_{[\epsilon\eta]} \phi_{\alpha}^{(1)[\beta\epsilon\eta]} + G_4^{(1)} \psi^{\alpha}_{[\beta\delta]} C \psi^{\epsilon}_{[\eta\xi]} \phi^{(1)[\beta\delta\eta\xi]}_{\{\alpha\epsilon\}} \quad (8)$$

and by analogous couplings in $L_Y^{(2)}$ for the composite fields constructed from right-handed preons $\psi_{\alpha m,R}$. The first coupling in Eq. (8) gives large ($\sim\mu$) masses to the states with the $8_R$ and $(5, 1)_L + (1, 3)_L$ components in the $8R + 216_L$ representation (and analogously in the $8_L + 216_R$ representation). The ordinary quarks receive their masses from the remaining couplings with the exotic scalars (the $u$, $c$, and $t$ "up"–quarks receive their masses from the last coupling in $L_Y^{(1)}$. The masses of the right-handed, quark-lepton families $(\bar{5}, \bar{3})_R + (10, \bar{3})_R$ in the $216_R$ representation are also formed from such couplings in $L_Y^{(2)}$. Since these exotic scalars (left-handed and right-handed preon scalars) at the same time give masses to weak bosons, the right-handed families, in comparison with the left-handed families, can be weighted primarily because of the Yukawa constants $G_{2,3,4}^{(2)}$ in the $L_Y^{(2)}$ couplings. Assuming that these constants are equal to $\sim 1$ (they are equal to $\sim 0.001$ and 0.01 in $L_Y^{(1)}$), we obtain a mass scale $m^{(R)} \sim (1\text{–}10) G_F^{-1} \sim 1$ TeV for three, right-handed families.

However, the fragments $(\overline{45}, 1)$, $(24, 3)$, $(1, 6)$, and $(5, 8)$ in the $216_L + 216_R$ representation, after all, remain massless. This is attributable to the absence of transitions of constituent, left-handed, preon fermions to right-handed preon fermions due to an exact symmetry $M$. Such transitions occur only if a singlet [under $SU(8)$] preon $\chi_{i,m}$ with a "left" and "right" metacolor is introduced in addition to the $\chi_{\alpha i,L}$ and $\tilde{\psi}_{\alpha m,R}$ preons. This preon is very heavy ($m_{\chi} \sim \mu$) and its metacolor interactions, in contrast to interactions of the fields, are vector rather than chiral interactions. It produces, together with the $\psi_{L,R}$ fields, a series of heavy, bound states of fermions and scalars. An eight-preon $(\widetilde{\chi\chi} \tilde{\psi}_L^{\alpha} \tilde{\psi}_L^{\beta} \tilde{\psi}_L^{\gamma} \psi_{\delta R} \psi_{\epsilon R} \psi_{\eta R})$, exotic scalar $\kappa^{\{[\alpha\beta]\gamma\}}_{\{[\delta\epsilon]\eta\}}$ in the Yukawa coupling $L_Y^{(12)} = G^{(12)} \overline{216}_L 216_R \kappa$ with large vacuum expectations ($\sim\mu$) in the $(200, 1)$ and $(1, 27)$ components, which weights the $(\overline{45}, 1)$, $(24, 3)$, $(1, 6)$, and $(5, 8)$ states, does not perturb the masses of the observed quarks, leptons and weak bosons.

In conclusion, we note that our theory of "metaforces" with a symmetry group $M = SO(3)_L \otimes SO(3)_R \sim SO(4)$ is not asymptotically free because of a large number of preons ($n > 10$), and the preon confinement is possible here only in the strong-coupling mode.[5] Such mode can be realized if we assume that the symmetry $M$ is an internal symmetry $SO(4)$ supergravitation.[6] In fact, we would have in this case large ($\sim$1) constants for the metacolor forces at Planck's distances. The structure of $M$ symmetry and the similarity of the range of forces between the preons and the gravitational range make this assumption quite plausible.

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1. S. G. Matinyan, Usp. Fiz. Nauk **130**, 3 (1980) [Sov. Phys. Usp. **23**, 1 (1980)].
2. J. C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1974).
3. A. A. Ansel'm, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 88 (1980) [JETP Lett. **31**, 80 (1980)].
4. G. 't Hooft, Cargese Lecture Notes, 1979.
5. K. G. Wilson, Phys. Rev. **10**, 2245 (1974); M. Creutz, Phys. Rev. Lett. **43**, 553 (1979).
6. A. Das, Phys. Rev. **D15**, 2805 (1977).

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