Stochastic particle motion and anomalous thermal conductivity in a tokamak

V. V. Parail and P. N. Yushmanov

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A self-consistent theory is derived for the spatial diffusion of electrons in electromagnetic fluctuations of arbitrary amplitude in a tokamak. Toroidal drift is taken into account. The diffusion is determined by banana electrons with a scale time $\tau \sim qR/v_e\sqrt{\epsilon}$ for the onset of a stochastic motion. The scale length of the diffusion which is nonlinear in the wave amplitude reaches saturation at a level $\sim 2c^2/\omega_p^2 + v_e^2q^2/\omega_{He}^2\epsilon$.

It has been shown experimentally that the energy loss by the electron mechanism in a tokamak is anomalously high. Most of the theoretical work on the mechanism for this anomalous electron thermal conductivity has been carried out in the quasilinear approximation.[1–3] It was suggested in Ref. 3 that a longitudinal Ohm's law be used for a self-consistent description of the oscillations of the electric and magnetic fields. This approach would make it possible to jointly consider the transport due to electric and magnetic fluctuations. In the same paper it was suggested that the thermal conductivity might reach saturation in a highly nonlinear regime. The quasilinear approximation which was actually used in Ref. 3, however, was not suitable for constructing a self-consistent theory for the anomalous transport in the nonlinear regime ($ck_\perp\varphi/B_0 = v_\perp > v_F = \omega/k_\perp$), which is the case of most interest for analyzing experimental results. In the present letter we offer the first systematic analysis at a nonlinear level of the anomalous electron transport in a tokamak in an interaction with electromagnetic fluctuations. The toroidal drift of electrons is taken into account.

Let us examine the motion of electrons in fluctuations of the electric field $\mathbf{E}_\perp = -\vec{\nabla}_\perp\varphi$ and the magnetic field $\mathbf{B}_\perp = [e_z\vec{\nabla}_\perp A_\parallel]$. We first analyze the transverse drift,

$$\frac{d\mathbf{r}_\perp}{dt} = \frac{c[\mathbf{e}_z\vec{\nabla}\varphi]}{B_0} + \frac{\mathbf{B}_\perp v_\parallel}{B_0} + \mathbf{v}_d. \tag{1}$$

We transform to a coordinate system which matches an oscillation localized near r_0 in the helical sense $[m = nq(r_0)]$. Considering in $v_\parallel$ only the unperturbed motion in the toroidal magnetic field of the tokamak, $v_\parallel(t) = v_0 + v_b\sin\omega_b t(\omega_b = v_e\sqrt{\epsilon}/qR)$, we rewrite Eq. (1) in Hamilton form,

$$d\mathbf{r}_\perp/dt = [\mathbf{e}_z\vec{\nabla}H], \tag{2}$$

where

$$H(r,\theta,t) = \frac{c}{B_0}\varphi(r,\theta,t) + \frac{v_\parallel(t)}{B_0}(A_\parallel(r,\theta,t) + A_s(r)) + v_d r\cos\theta(t).$$

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The term with A_s describes the motion caused by the change in the helical index of the lines of force with distance from the resonant surface. In accordance with the observed spectrum of oscillation frequencies, which are substantially lower than the frequency of the motion of the particles in their periodic orbits,[4] $\omega \leqslant 3\times 10^5 \ll \omega_b \sim 2\times 10^6$, the basic time dependence in Hamiltonian H is incorporated in $v_{\parallel}(t)$, and the time dependence of φ and $A_{\parallel}$ can be ignored. We first consider motion without a toroidal drift ($v_d = 0$); we can then write the drift Hamiltonian of the electrons as

$$H(r,\theta,t) = c\varphi(r,\theta)/B_0 + v_{\parallel}(t)A_{\parallel}(r,\theta)/B_0 . \qquad (3)$$

The first conclusion which we draw from the form of the Hamiltonian is that the motion of the untrapped particles, with $v_{\parallel}(t) = v_0$, is integrable and thus cannot lead to a radial diffusion. A radial diffusion could result from only the trapped particles, with $v_{\parallel}(t) = v_b \sin \omega_b t$, and then only if $c\varphi \lesssim v_b A_{\parallel}$, since at $c\varphi \gg v_b A_{\parallel}$ the motion of the electrons is along equipotentials $\varphi(r,\theta) = \text{const}$. The nature of the diffusion is also qualitatively clear from the form of Hamiltonian (3). At $c\varphi \lesssim v_b A_{\parallel}$, the particles move along contour lines which are averages of the contour lines of φ and $A_{\parallel}$. These average lines are different at different times. A measure of the spatial discrepancy of the paths in the case of relatively low amplitudes is the displacement over a time $\omega_b^{-1} \sim \varphi$, while a corresponding measure at high amplitudes ($\Omega \sim k_{\perp} v_{\perp} > \omega_b$) is the difference between the φ and $A_{\parallel}$ equipotentials. Since the oscillation amplitudes observed experimentally are quite large,[4] it is the second case in which we are primarily interested. Furthermore, only in this case can we expect the diffusion coefficient to be independent of the oscillation amplitude.

Assuming that only oscillations with a given helical index $m = nq(r)$ are excited near a rational surface, we write the potential φ as

$$\varphi(r,\theta,t) = \varphi(r)\exp(-i\omega t + im\theta + in\varphi). \qquad (4)$$

The relationship between $A_{\parallel}$ and φ is determined by Ohm's law, which follows from the longitudinal equation of motion of the electrons:

$$\frac{dv_{\parallel}}{dt} = \frac{e}{m_e}\frac{\mathbf{B}_0\vec{\nabla}\varphi}{B_0} + \frac{e}{m_e c}\left(\frac{\partial A_{\parallel}}{\partial t} + \frac{c}{B_0}[\vec{\nabla}\varphi\vec{\nabla}A_{\parallel}]\right). \qquad (5)$$

For use below, we use (4) to rewrite the first term on the right side of (5) in the form $em_e^{-1}k_{\parallel}\omega^{-1}\partial\varphi/\partial t$.

In large-amplitude oscillating fields ($\Omega \sim k_{\perp} v_{\perp} \gg \omega, k_{\parallel} v_{\parallel}$), the transverse motion of the particles is a motion along closed cells, as was shown above. Accordingly, the particles cannot undergo a displacement with respect to the wave along the poloidal angle in the case of a high oscillation level, in contrast with the situation in the case of a linear motion. The phase of a wave on a path in this case oscillates rapidly with respect to its average values: $\varphi(t) = \varphi_0 \exp(-i\omega t + iz \sin \Omega t)$, where $z \lesssim 1$. We thus see that the longitudinal thermal motion cannot substantially change the phase φ on a path, and it thus cannot affect the relationship between φ and $A_{\parallel}$. Using this result, we rewrite Eq. (5) as follows, assuming that the motion of the particles is along hydrody-

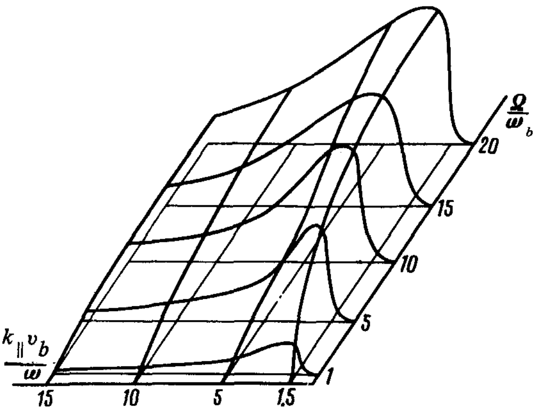

FIG. 1. The diffusion coefficient of the trapped electrons versus the parameter $k_{\parallel} v_b/\omega = k_{\parallel} v_e \sqrt{\epsilon}/\omega$ and versus the oscillation amplitude $\Omega/\omega_b \sim ck_{\perp}^2 \varphi/B_0\omega_b$.

namic paths:

$$\frac{dv_{\parallel}}{dt} = \frac{e}{m_e c}\left(\frac{\partial}{\partial t} + \frac{c}{B_0}[\mathbf{e}_z \vec{\nabla}\varphi]\vec{\nabla}\right)\left(A_{\parallel} - \frac{k_{\parallel} c}{\omega}\varphi\right). \tag{6}$$

It is easy to see that the operator $\partial/\partial t + cB_0^{-1}[e_z \vec{\nabla}\varphi]\vec{\nabla}$ on the right side of this equation is completely equivalent to the operator d/dt on the left side, so that the solution of Eq. (6) takes the simple form $v_{\parallel} = (e/m_e c)(A_{\parallel} - k_{\parallel} c\varphi/\omega)$. Using $j_{\parallel} = c\Delta_{\perp} A_{\parallel}/4\pi$, we immediately find the relationship between φ and $A_{\parallel}$:

$$-(c/\omega_p)^2 \Delta_{\perp} A_{\parallel} + A_{\parallel} = k_{\parallel} c\varphi/\omega. \tag{7}$$

This result is the same as the equation which follows from the linear Ohm's law for a plasma without a thermal motion of electrons. We see from (7) that the scale length of the difference between the contour lines of φ and $A_{\parallel}$ is c/ω_p.

This analysis of the electron thermal conductivity in nonlinear oscillations under the condition $v_d = 0$ thus reduces to an analysis of the drift motion of the particles which is described by Hamiltonian (3) with relationship (7). Figure 1 shows the results of numerical calculations of D as a function of the parameter $k_{\parallel} v_b/\omega$ and of the oscillation amplitude (the frequency ratio Ω/ω_b). We see that in the limit of a pronounced nonlinearity of the oscillations ($\Omega \gtrsim \omega_b \gg \omega$), the diffusion coefficient reaches the asymptotic value $D \sim 2\omega_b c^2/\omega_p^2$ and becomes independent of the fluctuation amplitude. Since only the trapped particles participate in the diffusion, the electron thermal conductivity can be estimated as $2\epsilon(c/\omega_p)^2 v_e/qR$.

Now turning to Eq. (1), we examine the consequences of incorporating a vertical drift of electrons. The functional time dependence of the term with v_d is the same as that for the term with $A_{\parallel}$. However, the former term leads to a different scale length for the onset of a stochastic motion: $\Delta x \sim v_d/\omega_b$. Our numerical calculations confirm this qualitative conclusion regarding the diffusion of the banana electrons.

The two mechanisms for the onset of a stochastic drift motion discussed by us here are independent, so that the resultant anomalous electron thermal conductivity can be estimated to be

$$\chi_e \sim (2\epsilon c^2/\omega_p^2 + v_e^2 q^2/\omega_{He}^2) v_e/qR.$$

The last term in this expression may determine the experimentally observable soft limit on β.

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