Stochastic ion diffusion in a tokamak

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A mechanism for the stochastic diffusion of ions in a tokamak as they interact with large-amplitude electromagnetic fluctuations is analyzed. The maximum anomalous-diffusion coefficient of the ions is shown to be close to the neoclassical ion thermal conductivity in the plateau regime.

Experiments on the ion thermal conductivity in a tokamak have shown that it is comparable in magnitude to the neoclassical value. On the other hand, the theoretical and experimental values of the ion thermal conductivity differ by a factor of several units, and this difference tends to increase in the region of low collision rates. This discrepancy might arise from either a significant size of the ion banana orbits[1] (significant in comparison with the scale dimension of the inhomogeneity) or a motion of the ions in fluctuations of the electromagnetic field. Here we will consider only the latter mechanism—an anomalous transport mechanism.

The anomalous transport of ions in fluctuations of an electromagnetic field was studied theoretically in Refs. 2 and 3, where it was shown that the anomalous ion transport is much lower than the electron transport. However, those studies were carried out under the assumption that the interaction of the particles with the waves can be described in the theory of weak turbulence. Under actual experimental conditions, the level of the electromagnetic fluctuations is quite high, so that the velocity of the particles in the fluctuations, $\tilde{v}$, exceeds the transverse wave velocity $v_F = \omega/k_\perp$. Consequently, the theory of weak turbulence is definitely inapplicable. A renormalization approach, which incorporates the motion of the particles in fluctuations in even the zeroth approximation, was taken in Ref. 4 in a study of the anomalous transport. The renormalization approach makes it possible to construct a self-consistent picture of the electron thermal conductivity in a tokamak which agrees satisfactorily with the scalings observed experimentally. In the present letter we take the same approach to analyze the anomalous ion transport.

Under actual conditions with a high oscillation amplitude $k_\perp \tilde{v} > \omega$, the drift motion across the unperturbed magnetic field is the basic motion, as was shown in Ref. 4:

$$d\mathbf{r}_\perp/dt = c[\mathbf{e}_z \vec{\nabla}\tilde{\varphi}]/B_0 + v_\parallel \tilde{\mathbf{B}}_\perp/B_0 + \mathbf{v}_d. \tag{1}$$

The first two terms correspond to drift in fluctuations of the electric and magnetic fields, while the last term corresponds to a toroidal drift, $v_d = (v_\parallel^2 + v_\perp^2/2)/\omega_B R$. In describing the motion of the ions we should take into account the circumstance that the motion in a perturbation of the magnetic field is unimportant because of the small thermal velocity $v_\parallel$, and this motion can be ignored. Furthermore, the drift in the potential $\tilde{\varphi}$ should be modified to allow for the finite Larmor radius of the ion. Consequently, in Eq. (1) we have not the local value of the potential, $\tilde{\varphi}(r_\perp)$ but an average

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over the Larmor orbit:

$$\bar{\varphi}(r_\perp, t) = (2\pi\rho(t))^{-1} \int \tilde{\varphi}(r'_\perp, t)\delta(\rho(t) - |r'_\perp - r_\perp|)\, dr'_\perp \ , \qquad (2)$$

where $\rho(t) = v_\perp/\omega_B \approx \rho_0(1 + (\epsilon/2)\sin \int v_\parallel dt/qR)$ is the time-dependent Larmor radius, $\epsilon = r/R$ is the ratio of the minor and major radii of the torus, and q is the safety factor. As in Ref. 4, we assume that the oscillations are stretched out markedly along the magnetic field ($E_\parallel \approx 0$), so that the longitudinal motion of the ions is unperturbed: $v_\parallel = (2/m)^{1/2}(E - \mu B)^{1/2}$.

To analyze the transverse transport, we rewrite Eq. (1) in Hamilton's form,

$$d\mathbf{r}_\perp/dt = [\mathbf{e}_z \vec{\nabla} H], \qquad (3)$$

where $H(r_\perp, t) = c\bar{\varphi}(r_\perp, t)/B_0 + rv_d \cos\theta(t)$ or, in the spectral representation ($\tilde{\varphi} = \Sigma_k \tilde{\varphi}_k$),

$$H(r_\perp, t) = cB_0^{-1} \sum_k \tilde{\varphi}_k(r_\perp, t) J_0(k_\perp \rho(t)) + rv_d \cos\theta(t). \qquad (4)$$

In a motion of this sort, a diffusion of particles can, of course, arise only by virtue of a time dependence $H(t)$. The dependence $\tilde{\varphi}_k(t)$ is characterized by the fluctuation frequency. In a tokamak, most of the oscillation energy is at low frequencies,[5] $\omega \lesssim 3\times 10^5$. The time dependence $\rho(t)$ and the time dependence $\theta(t)$ have a characteristic frequency $\omega_{bi} \approx v_\parallel/qR \sim 3\times 10^5\ \mathrm{s}^{-1}$. We will ignore the dependence $\tilde{\varphi}_k(t)$ here; incorporating it could not substantially change the transport coefficient found below, because of the condition $\omega \lesssim \omega_{bi}$.

We first consider the motion of ions without a toroidal shift ($v_d = 0$). Under the condition $k_\perp \rho\epsilon \ll 1$, we can carry out an expansion in Hamiltonian (4) and write it in the form

$$H(r_\perp, t) = \frac{c}{B_0} \sum_k \tilde{\varphi}_k(r_\perp)\left[J_0(k_\perp\rho_0) + \frac{k_\perp\rho_0\epsilon}{2}\sin(\omega_{Bi}t)J_1(k_\perp\rho_0)\right]. \qquad (5)$$

If we ignore terms proportional to $J_1(k_\perp\rho_0)$, we see that the motion occurs along equipotentials of H, without diffusion. The time-dependence term in H causes the equipotentials to change in shape over time. At large oscillation amplitudes ($\Omega = k_\perp \tilde{v} > \omega_{bi}$), the scale length of the diffusion should be independent of the oscillation amplitude, while it will be determined by a measure of the spatial separation of the equipotentials at different times. This separation occurs over a scale for which the condition $k_\perp\rho\epsilon \gtrsim 1$ holds; i.e., $\Delta x \sim \rho\epsilon$. Consequently, in the limit of large fluctuation amplitudes, the transport coefficient can be estimated to be $D \sim \epsilon^2\rho^2 v_i/qR$.

Incorporating the toroidal drift gives rise to an additional, time-dependent term in the Hamiltonian. This additional term does not depend on the fluctuation amplitude, so that a significant distance between equipotentials, on the order of $\Delta x \sim q\rho$, occurs only at a certain fluctuation amplitude, at which the relation $v \sim v_d$ holds. In both the case $\tilde{v} \gg v_d$ and the case $\tilde{v} \ll v_d$, the distance between the paths and thus the diffusion coefficient will decrease. The maximum diffusion coefficient (at $\tilde{v} \sim v_d$) can be estimated from

$$D \sim v_d^2 / \omega_{bi} \sim q\rho^2 v_i / R. \qquad (6)$$

To test these arguments regarding the nature of the stochastic diffusion of ions, we have carried out some numerical Monte Carlo calculations of the diffusion coefficient. Analysis of the results shows that the maximum diffusion coefficients for both mechanisms are close to the analytic estimates. The results also confirm the dependence of the diffusion coefficient on the fluctuation amplitude when toroidal drift is taken into account, with a maximum being reached at $\tilde{v} \sim v_d$. A detailed analysis of the dependence of the diffusion coefficient on the plasma parameters requires extensive computations, which go beyond the scope of the present letter.

Let us briefly discuss the mechanism which limits the fluctuation amplitude. If we assume that the condition $v_d \ll \tilde{v}$ holds, then all the ions will move along cells formed by the potential $\tilde{\varphi}$, undergoing slight excursions from the equipotentials as a result of the vertical drift. Since the displacement of all the ions in a cell occurs in the same direction, the ion charge becomes redistributed, and it distorts the potential of the cell. An estimate of the potential which results is very high, $e\varphi / T \sim M/m$, so that only a small fraction of the ions can be trapped in cells. We thus immediately have a restriction on the oscillation amplitude, at a level, $\tilde{v} \sim ck_\perp^2 \tilde{\varphi} / B_0 < v_d$, such that the stochastic diffusion coefficient is close to the maximum value, given in (6).

In summary, this study has shown that the nonlinear ($\tilde{v} > \omega / k_\perp$) electromagnetic fluctuations which exist in a tokamak plasma give rise (as do electrons) to a stochastic diffusion of ions. The magnitude of the anomalous ion diffusion (and that of the thermal conductivity) is close to the neoclassical thermal conductivity in the plateau regime. The polarization of the plasma, which is caused by motion in fluctuations, may serve as a mechanism for saturation of the oscillation amplitude in a tokamak.

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