Fluctuation properties of small conductors

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A theory is derived for fluctuations of the conductivity in a regime of weak localization. These fluctuations are shown to determine the dependence of the properties of miniature samples on external fields and also the anisotropy of these properties.

1. The properties of macroscopic samples—the electrical conductivity, the specific heat, the magnetic susceptibility, etc.—are determined not by the particular realization of a random potential but by simply a small number of microscopic characteristics such as the dimensions and shape of the sample, the impurity concentration, etc. The properties of samples with identical macroscopic characteristics do differ, but the difference is inversely proportional to the square root of the sample volume v. Just how large a sample has to be in order to be "big enough" depends on the particular mechanism which leads to the self-averaging of the characteristics. It turns out that the characteristic dimension L_c may be many orders of magnitude larger than the interatomic distance, and at $T=0$ it becomes infinite.[1] Samples with $L<L_c$ are not macro-

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scopic; their properties are determined not only by the macroscopic characteristics, and they vary from sample to sample by amounts which are independent of the dimensions. Such samples and their properties, which are determined by fluctuations, we call "mesoscopic," following Ref. 2. A theory of mesoscopic properties must be statistical and must predict the distribution function, e.g., of the conductivity, or its moments (the conductivity averaged over realizations, its dispersion, etc.), not the resistance of a particular sample. We are especially interested in effects which are not seen in macroscopic objects: mesoscopic effects. Some of these effects have already been predicted qualitatively[3] (see also Ref. 2) and have recently been discovered experimentally.[4,5] In the present letter we propose a theory of mesoscopic effects at low temperatures in samples which have a good metallic conductivity, for mean free paths $l \gg \lambda$, where $\lambda = \hbar/p_F$ is the de Broglie wavelength of the electron.[1)]

2. The conductivity G of a mesoscopic sample is a random function of the external parameters, e.g., the magnetic field H: $G(H)$. This functional dependence is characterized by the correlation function

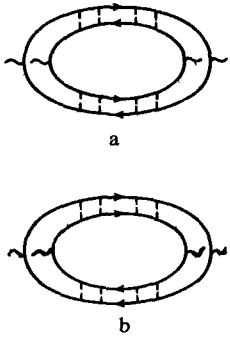
$$F(H, \Delta H) = \langle G(H)\, G(H + \Delta H)\rangle - \langle G(H)\rangle^2 . \tag{1}$$

Here the angle brackets denote an average over the realizations of the random potential. The quantity $\sqrt{F(H,0)}$ tells us the characteristic amplitude of the nonmonotonic changes in $G(H)$, while the value ΔH, over which there is a substantial change in $F(H,\Delta H)$, determines the characteristic value of the magnetic field, H_c, for the function $G(H)$.

The function $F(H,\Delta H)$ can be calculated by the method of Refs. 1 and 6. Figure 1, a and b, shows Feynman diagrams corresponding to the expression

$$F(H, \Delta H) = \frac{4\alpha}{\upsilon} S \left(\frac{G}{\nu}\right)^2 \int d\epsilon d\epsilon' \frac{\partial n}{\partial \epsilon} \frac{\partial n}{\partial \epsilon'} \int d\mathbf{r}$$
$$\times \{ |P^{(D)}_{\epsilon - \epsilon'}(0, \mathbf{r})|^2 + |P^{(C)}_{\epsilon - \epsilon'}(0, \mathbf{r})|^2 \} , \tag{2}$$

where ν is the state density, G is the conductivity of the sample, α_S is a coefficient[1] which depends on H, and $P^{(C)}_{\omega}$ ($P^{(D)}_{\omega}$) is a cooperon (diffusion), which satisfies the



a

b

FIG. 1.

equation

$$\left\{-i\omega + D\left(-i\hat{\partial}_{C,D} + \frac{e}{c\hbar}\Delta\mathbf{A}\right)^2 + \frac{1}{\tau_{in}}\right\} P_\omega^{(C),(D)}(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}') . \quad (3)$$

Here D is the electron diffusion coefficient, $\partial_D = \nabla$, $\partial_C = \nabla + 2ie\mathbf{A}/c\hbar$, rot $\mathbf{A} = \mathbf{H}$, rot $\Delta\mathbf{A} = \Delta\mathbf{H}$, and τ_{in} is the scale time between inelastic collisions.

The additional term $e\Delta\mathbf{A}/\hbar c$ arises in (3) because one of the electron lines on the diffusion (cooperon) ladders in Fig. 1 corresponds to the magnetic field H, while the other corresponds to $H + \Delta H$. Consequently, if only the cooperon depends on H, then ΔH also affects the diffusion.[2)] It follows from (2) and (3) that in a ring with an average radius R the quantity F (first) oscillates as a function of the magnetic flux $\Delta\phi = \pi R^2\Delta H$ and (second) decreases with increasing ΔH. The first consequence leads to regular oscillations of the function $G(\Delta\phi)$ with a period $\phi_0 = 2\pi\hbar c/e$, while the second consequence leads to an irregular and nonmonotonic change in $G(H)$ not only in the ring but also in singly connected regions. The characteristic magnitude of this change is $H_c \sim \phi_0/S$, where S is the area of the sample. Oscillations with a period ϕ_0 and irregular variations in $G(H)$ have been observed experimentally.[4,7] Regular oscillations with a period ϕ_0 have a random phase and therefore disappear when G is averaged over the realizations of the random potential. The quantity $\langle G\rangle$ can oscillate only with a period of $\phi_0/2$ because of the H dependence of the cooperon, and this oscillation is always accompanied by a monotonic dependence[8] $\langle G(H)\rangle$.

The amplitude of the conductivity fluctuations depends on the relations among T, $D\hbar/L_i^2$ (L_i are the linear dimensions of the sample, $i = x,y,z$) and $\hbar/\tau_{\text{in}}$. Carrying out the integration over ϵ and ϵ' in (2), we find, within a numerical factor,

$$F(\Delta H = 0) \sim \left(\frac{e^2}{\hbar}\right)^2 \frac{1}{L_x^3} \begin{cases} L_z L_y L_T, & d = 3\,(L_T < L_i) \\ L_y L_T^2, & d = 2\,(L_z < L_T < L_{y,x}) \\ L_T^2 L_\epsilon, & d = 1\,(L_{y,z} < L_T < L_x). \end{cases} \quad (4)$$

Here L_x is the dimension of the sample along the direction of the measurement current; $L_T = \sqrt{D\hbar/T}$; $L_\epsilon = min\{\sqrt{D\tau_{\text{in}}}; L_x\}$; and $F \sim (e^2/\hbar)^2$ at $L_i \ll L_T$. The amplitude of the regular oscillations of $G(\phi)$ with a period ϕ_0 in the ring decreases exponentially with increasing T.

3. Macroscopic samples, despite the presence of a random potential in them, are symmetric under rotations and reflections. In the case of mesoscopic samples, on the other hand, they do not have these symmetries, and in them we should see effects which are characteristic of anisotropic substances and of substances without an inversion center. Let us examine some of these effects.

In a zero magnetic field, a voltage of random magnitude and sign, directed perpendicular to the measurement current, arises. The conductivity correlation function is

$$F_{xi,\,xj} = \langle G_{xi} G_{xj} \rangle - \langle G_{xi} \rangle \langle G_{xj} \rangle = F \frac{L_i L_j}{L_x^2} \delta_{ij}. \tag{5}$$

As the magnetic field is changed, G_{xy} varies in a nonmonotonic way over the same characteristic field intervals H_c as G_{xx}, but G_{xx} and G_{xy} are not correlated: $\langle \delta G_{xx} \delta G_{xy} \rangle = 0$.

The absence of an inversion center should lead to a huge generation of the second harmonic in granulated samples subjected to light, as has recently been observed in island films.[9] Another example of effects of this kind, which is apparently amenable to experimental observation, is the ability of mesoscopic samples to rectify an alternating current, i.e., the dependence of the conductivity G on the sign of the potential difference V. If[3)] $E_c = D\hbar/L_x^2 \gg eV$[3)], then

$$W(V) = \langle [G(V) - G(-V)]^2 \rangle \simeq \left(\frac{e^2}{\hbar} \frac{eV}{E_c} \right)^2 \tag{6}$$

Expression (6) can be derived from (2) by incorporating an electric field in Eq. (3) for the cooperons (or diffusions).

We have mentioned only a few of the many unusual phenomena which should occur in miniature samples at low temperatures. The scale size of the conductivity fluctuations here is on the order of $e^2/\hbar \simeq 10^{-4}$ S. At a temperature $T \simeq 0.05$ K and with a diffusion coefficient $D \simeq 50$ cm^2/s, mesoscopic effects have been observed in samples with dimensions less than a micron.[4] Under these conditions we have $E_c \simeq 10^{-5}$ eV. Mesoscopic effects merit a comprehensive theoretical and experimental study.

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[1)] After this paper had been prepared for publication, we received a preprint from Lee and Stone,[6] with an independent derivation of the results of Ref. 1 and of some of the results of the present letter.

[2)] It is of fundamental importance to incorporate both the cooperon and diffusion contributions. If one of them is ignored, for example, the identity $\langle [G(H) - G(-H)]^2 \rangle = 0$, which follows from the Onsager relations, is violated.

[3)] The conductivity of field-effect transistors depends nonmonotonically on the chemical potential (or the gate voltage V_g).[5] The characteristic value of this dependence is on the order of E_c (Ref. 6).

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