Normal Fermi liquid in a superfluid ^{3}He-A at $T=0$ and the anomalous current

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In the presence of a texture in the field of an orbital vector **l**, the subsystem of a superfluid ^{3}He-A is a normal Fermi liquid which consists of particles at the zero Landau level in an effective "magnetic" field produced by the texture. In the ground state, the Fermi liquid carries an anomalous current (parallel to the magnetic field)—$(\hbar/2m_3)\,\rho\mathbf{l}\,(\mathbf{l}\,\mathrm{curl}\,\mathbf{l})$ which plays a decisive role in the dynamics of ^{3}He-A at $T=0$.

The expression for the current in ^{3}He-A at $T=0$ contains an anomalous term

$$\mathbf{j}_{\rm an} = -\frac{\hbar}{2m_3} C_0 \mathbf{l}\;(\mathbf{l}\,\mathrm{curl}\,\mathbf{l}), \qquad (1)$$

which does not appear in superfluid hydrodynamics at $T=0$ because it violates the Galilean invariance and momentum conservation (in the weak-coupling approximation, the parameter C_0 is the same as the density of the liquid, ρ). To solve this paradox, Volovik and Mineev[1] have postulated the existence of a normal component of ρ_n in ^{3}He-A at $T=0$ consisting of excitations which can accumulate in the texture even at $T=0$ by virtue of the gap-free spectrum of ^{3}He-A with $\mathbf{k}\|\mathbf{l}$.

This postulate was proved by Combescot and Dombre,[2,3] who showed in the first place that near $\mathbf{k}\|\mathbf{l}$ the excitation spectrum of a texture is similar to the spectrum of a charged particle in a magnetic field. As a result, the state density at the Fermi surface is nonvanishing, since it is the same as the state density at the zero Landau level.[2] This circumstance accounts for the existence of $\rho_n\ (T=0)$, which is in order-of-magnitude agreement with the estimate given in Ref. 1. Secondly, analyzing the zero-energy excitation dynamics, they found that the normal component incurs the superfluid-momentum deficit, thereby restoring the momentum conservation and the Galilean invariance.[3]

Combescot and Dombre[3] hypothesized that the anomalous current is linked with the normal motion rather than the superfluid motion, which changes the collisionless dynamics of ^{3}He-A at $T = 0$. We will show that this hypothesis is correct: The states at the zero Landau level form a normal (nonsuperfluid) Fermi liquid which has the current (1) in the ground state. This current, which is directed along the magnetic field, resembles the chiral anomaly in the gauge theories.[4]

To prove this assertion, we will find the eigenstates and eigenfunctions of the quasiparticles in ^{3}He in the presence of a texture in a field $\mathbf{l}$. Following the procedure used in Ref. 3, we will use the Bogolyubov equations for a two-component wave function $\psi = (u, v)$ of the Bogolyubov quasiparticle

$$\hat{H}\psi = E\psi, \quad \hat{H} = \begin{pmatrix} \frac{\mathbf{p}^2}{2m_3} - \mu & \frac{1}{2k_F}(\vec{\Delta}\hat{\mathbf{p}} + \hat{\mathbf{p}}\vec{\Delta}) \\ \frac{1}{2k_F}(\vec{\Delta}^*\hat{\mathbf{p}} + \hat{\mathbf{p}}\vec{\Delta}^*) & \mu - \frac{\hat{\mathbf{p}}^2}{2m_3} \end{pmatrix}. \tag{2}$$

Because we are concerned with only the current-conducting vacuum state, we will not restrict the analysis, in contrast with the approach in Ref. 3, to the frequencies near the Fermi surface. Here $\hat{\mathbf{p}} = (1/i)\nabla$, μ is the chemical potential, and $\boldsymbol{\Delta}$ is the orbital part of the order parameter: $\vec{\Delta} = \Delta_0[\mathbf{e}_1(\mathbf{r}) + i\mathbf{e}_2(\mathbf{r})]$, where $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{l} = [\mathbf{e}_1\mathbf{e}_2]$ are unit vectors. For simplicity, we consider the texture, in which curl $\mathbf{l} \| \mathbf{l}$, and we expand the order parameter near the origin, where the unit vectors run along the $\hat{x}$, $\hat{y}$, $\hat{z}$ axes of the Cartesian coordinates:

$$\mathbf{e}_1(\mathbf{r}) + i\mathbf{e}_2(\mathbf{r}) \approx \hat{\mathbf{x}} + i\hat{\mathbf{y}} - i\hat{\mathbf{z}}Bx, \qquad B = (\mathbf{l}\,\mathrm{curl}\,\mathbf{l})|_{\mathbf{r}=0}. \tag{3}$$

Using the Pauli matrices σ_i, we find the following expression for Hamiltonian (2):

$$\hat{H} = \sigma_3\left(\frac{\hat{\mathbf{p}}^2}{2m_3} - \mu\right) + \frac{\Delta_0}{k_F}\left(\sigma_1\hat{p}_x + \sigma_2(\hat{p}_y - \hat{p}_zBx)\right). \tag{4}$$

The eigenfunctions of this Hamiltonian are

$$\psi = e^{ik_zz + ik_yy}\,\Phi(x),$$

where Φ is the eigenfunction of the equation

$$\left\{\sigma_3\epsilon + \frac{\Delta_0}{k_F}\left(\sigma_1\frac{1}{i}\frac{\partial}{\partial x} + \sigma_2(k_y - k_zBx)\right)\right\}\Phi = E\Phi. \tag{5}$$

Equation (5) is equivalent to the Dirac equation for the motion of a charged particle in a magnetic field, which is $\hat{z}k_zB$. The spectrum of such a particle is

$$E_{n,\,k_z} = \left[\epsilon^2 + 2n\,|Bk_z|\left(\frac{\Delta_0}{k_F}\right)^2\right]^{1/2} \operatorname{sign}\epsilon\,, \tag{6}$$

where n is the number of the Landau level. The sign of the root is chosen in such a way that $E \to \epsilon$ for large values of $|\epsilon|$. For small values of B, we assume $\epsilon = (k_z^2/2m_3) - \mu$, since $k_y^2 \sim \langle \hat{p}_x^2 \rangle \sim k_z B$. The wave functions depend on the sign of the field $k_z B$ and for $B > 0$ they can be written

$$\Phi_n(k_z < 0) = \begin{pmatrix} u_n f_n(\tilde{x}) \\ i v_n f_{n-1}(\tilde{x}) \end{pmatrix}, \qquad \Phi_n(k_z > 0) = \begin{pmatrix} u_n f_{n-1}(\tilde{x}) \\ i v_n f_n(\tilde{x}) \end{pmatrix}, \tag{7}$$

where $f_n(\tilde{x})$ are the eigenfunctions of the harmonic oscillator, $\tilde{x} = x - k_y/Bk_z$, and $f_{-1} = 0$. The parameters u_n and v_n are

$$2u_n^2 = 1 + \frac{\epsilon}{E_n}, \quad 2v_n^2 = 1 - \frac{\epsilon}{E_n}. \tag{8}$$

We notice that the state with $n = 0$ is missing at $k_z > 0$, since $v_0 = 0$, according to (8) and (6). Otherwise, the energy spectrum is symmetric with respect to the substitution $k_z \to -k_z$ (see Fig. 1).

Each Landau level n corresponds to its own Fermi filling, i.e., its own Fermi liquid. A Fermi liquid with $n \neq 0$ has a gap, $\Delta_0(2n|B|/k_F)^{1/2}$, in the spectrum and hence is superfluid, in contrast with a Fermi liquid with $n = 0$, which is normal. The momentum of this normal subsystem is not in equilibrium, because it contains particles with only a negative k_z. Working from the assumption that the state density at the

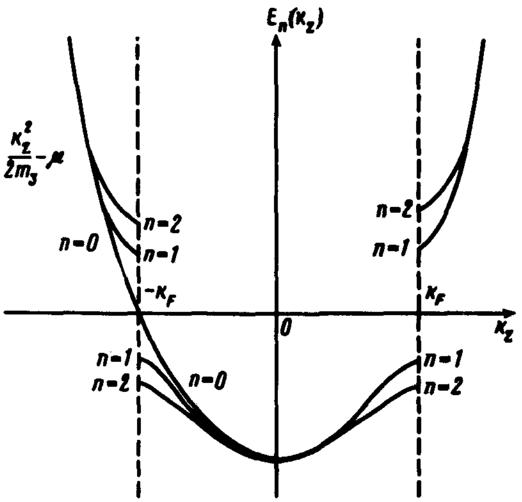

FIG. 1. Energy spectrum of the Bogolyubov quasiparticles in the ^{3}He-A texture [see Eq. (6)] for several Landau levels. The momenta of the particles at the zero level are pointing in the same direction ($k_z < 0$) and thus produce anomalous current (1).

zero Landau level, $\nu(\epsilon) = |B|/2\pi^2$, is independent of ϵ, we find the current which the normal ground-state Fermi liquid carries: This current, directed along the magnetic field, is

$$\mathbf{j} = \hat{\mathbf{z}} \int_{k_z < 0} k_z \nu(\epsilon)\, d\epsilon = - \hat{\mathbf{z}} \frac{k_F^3}{6\pi^2} B = - \frac{\hbar}{2m_3} \rho \mathbf{l}\, (\mathbf{l}\, \mathrm{curl}\, \mathbf{l}). \tag{9}$$

The current (1), which is not incorporated into the hydrodynamics of the superfluid component, is thus actually carried by the normal subsystem, which also accounts for the nonzero density of the normal component $\rho_n = \nu(0) k_F^2$ at $T = 0$.

We wish to emphasize the analogy with the chiral anomaly in the field theories. The current directed along the magnetic field will not arise if we work from the symmetry of Hamiltonian (5). The replacement $k_z \to -k_z$ is equivalent to reversing the field sign, which cannot change the classification of the levels. The anomalous current appears because the vacuum breaks the symmetry: The reversal of the field sign must be accompanied by a change in the direction of the Bogolyubov "spin", i.e., by a replacement of particles by holes. The vacuum consisting of the Fermi filling of particles is, however, not invariant under this transformation. The asymmetry of the spectrum in the texture has also been analyzed by Ho *et al.*[5]

The carrying of the anomalous current by the normal subsystem has the following effect on the collisionless ($\omega\tau > 1$) dynamics of ^{3}He-A at $T = 0$, which seems to be applicable at all frequencies $\omega \neq 0$, since the time of flight $\tau(T \to 0) \to \infty$. In the collisionless regime, the normal subsystem does not have time to keep track of the superfluid subsystem, so that current (1) remains constant over time (at least in the case of the given texture). The parameter C_0 is therefore a dynamic invariant $(\partial/\partial t) C_0 = 0$, consistent with the result of Ref. 6, which was obtained both phenomenologically and by the method of linear response of C_0 to an external perturbation.

Incorporation of the next higher orders in B shows that a subsystem with $n = 0$ forms a gap in the spectrum linear in B (in contrast with the gap $\sim\sqrt{B}$ that forms at $n \neq 0$). This gap is a characteristic frequency which distinguishes the hydrodynamic (low-frequency) regime from the collisionless (high-frequency) regime at $T = 0$ when the subsystem with $n = 0$ either can keep track of the rest of the system $[C_0(t) = \rho(t)]$ or it cannot keep track of it, $(\partial/\partial t) C_0 = 0$. Because of the gap in the spectrum, the density of the normal component vanishes at the lowest possible flow velocities of the liquid and then returns to its original value at the critical velocity which is proportional to the gap.

A comparison with the results of Ref. 7 shows that the normal subsystem can also be treated as a system of particles that fill the core of an irremovable vortex which is found in ^{3}He-A in the phase Φ of the order parameter: $\Delta(\mathbf{r})\mathbf{k} = |\Delta(\mathbf{r})\mathbf{k}| e^{i\Phi(\mathbf{k},\mathbf{r})}$.

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