Underground cascades in muon groups: new method for determining the chemical composition of the primary cosmic rays

V. N. Bakatanov, S. N. Boziev, A. V. Voevodskiĭ, Yu. F. Novosel'tsev,
M. V. Novosel'tseva, Yu. V. Sten'kin, and A. E. Chudakov
Institute of Nuclear Research, Academy of Sciences of the USSR

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Data from the underground scintillation telescope at the Baksan Neutrino Observatory are used to propose a new experimental method for determining the average atomic number of the primary nuclei in the energy range 10^{12}–10^{14} eV/nucleon. The first results, from 7500 h of observation, are reported.

1. The large effective area and the calorimetric properties of the telescope of Refs. 1–3 make it possible to simultaneously measure the energy (E_c) of a cascade induced by a muon in the material of the apparatus or in the surrounding soil and the number (m) of muons with energies $E_\mu \geqslant 220$ GeV accompanying the cascade. By considering the mechanism by which the cascades are formed by muons, the energy lost by the muons as they pass through the soil above the apparatus, and the production of pions in the first interaction event of the primary nucleons, we can analytically determine the spectrum of primary nucleons which are responsible for a cascade with an energy E_c. We should point out that, in contrast with experiments on extensive air showers, where the energy of the primary nucleus is determined from the total number of particles in the shower, in the present case we are reconstructing the energy of only one of the nucleons of the nucleus.

Correcting m for the effective fraction (Δ) of the muons that are detected by the apparatus, and taking into account the loss of a fraction (c) of the muons in the apparatus "occupied" by the cascade, we can find the average number of muons (N_μ) as a function of the primary energy (E_0). Assuming that we can apply a superposition principle to the nucleons of the primary nucleus (A_i) in their interaction with nuclei of the air, and comparing the resulting dependence $N_\mu = \overline{A} f(E_0)$ with the theoretical dependence $[N_0 = f(E_0)]$, we can draw a conclusion about the average atomic number of the primary nuclei: $\overline{A} = N_\mu / N_0$.

2. Specifying the underground muon energy spectrum to be $P(\epsilon_\mu) = [220 + \epsilon_\mu)^{-\gamma_\mu - 1}$, where $\gamma_\mu = 2.7$ is the exponent of the integral muon spectrum at the surface, and ϵ_μ is expressed in gigaelectronvolts, we can find the spectrum of the muons which are responsible for a cascade of energy E_c:

$$P_1(\epsilon_\mu, E_c) = P(\epsilon_\mu) W_\mu(\epsilon_\mu, E_c),$$

where $W_\mu(\epsilon_\mu, E_c)$ is the probability that a muon with energy ϵ_μ will initiate a cascade with an energy E_c through bremsstrahlung, the production of δ-electrons, the production of e^+-e^- pairs, and photonuclear interactions. The cross sections for these processes are taken from Refs. 4–7, respectively. Taking into account the energy loss of the muons as they traverse a layer of material of thickness x (we are ignoring the fluctuations of the loss), we can then write the following expression for the energy spectrum at the surface of the muons which are responsible for a cascade of energy E_c:

$$P_2(E_\mu, E_c) = \exp(bx) P_1 \left\{ \exp(bx) \left[E_\mu - \frac{a}{b} \left(\exp(bx) - 1 \right) \right], \; E_c \right\},$$

where $a = 2$ MeV·cm^2/g, and $\exp(bx) = 0.7$.

Correspondingly, the spectrum of the pions which are responsible for a cascade with energy E_c (where we are making use of the equality of the number of muons and the number of decayed pions) can be written as follows:

$$P_3(E_\pi, E_c) = \int_{nE_\pi}^{E_\pi} \left[\frac{\gamma_p + 1}{1 - n^{\gamma_p + 1}} + \frac{\gamma_p + 2}{1 - n^{\gamma_p + 2}} \frac{E_\mu \cos\theta}{B} \right] \frac{E_\mu^{\gamma_p + 1} P_2(E_\mu, E_c) dE_\mu}{E^{\gamma_p + 2} \left(1 + \frac{E_\pi \cos\theta}{B} \right)},$$

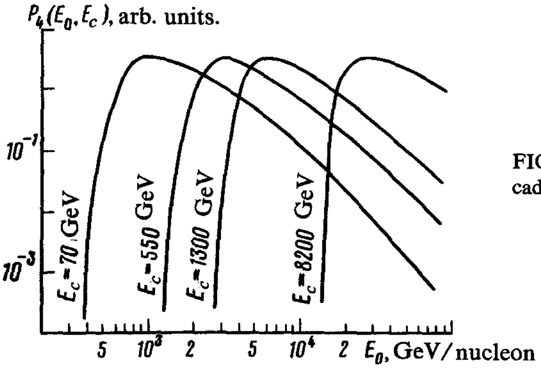

FIG. 1. Spectra of nucleons responsible for cascades with E_c.

where $B = 110$ GeV, $\gamma_p = 1.7$ is the exponent of the integral energy spectrum of the primary nucleons, $\theta = 35°$ is the average zenith angle, $n = (m_\mu/m_\pi)^2$, m_μ and m_π are the rest masses of the muon and pion, respectively, and the spectrum of nucleons responsible for the cascade with energy E_c is

$$P_4(E_0, E_c) = E_0^{\gamma_p - 1} \int P_3(E_\pi, E_c) W_\pi(E_0, E_\pi) dE_\pi ,$$

where $W_\pi(E_0, E_\pi)$ is the structure function of the production of charged pions in inclusive *pp* interactions.[8] Figure 1 shows the spectra of nucleons responsible for the cascades E_c. The average energies for spectra P_1–P_4, corresponding to energies E_c averaged over the specified ranges of E_c, are listed in Table I.

3. Table I shows the average muon multiplicities m corresponding to the average values $\overline{E}_c$. The total number of muons in a group can be reconstructed after allowing for two effects. (a) The apparatus detects only a fraction (Δ) of the total number of muons because of its finite dimensions. As the cascade energy increases, the fraction Δ also increases, because the axis of the muon group is "attracted" to the center of the apparatus. This attraction of the axis results from an increase in the energy of the

TABLE I. Energies in TeV.

E_c	from	0.05	0.10	0.21	0.44	0.74	1.18	1.47	2.94	6.62
	to	0.10	0.21	0.44	0.74	1.18	1.47	2.94	6.62	—
$\overline{E}_c$		0.07	0.14	0.29	0.55	0.90	1.30	1.91	3.38	8.21
$\overline{\epsilon}_\mu$		0.43	0.54	0.79	1.25	1.89	2.60	3.73	7.26	14.9
$\overline{E}_\mu$		0.84	1.00	1.36	2.00	2.94	3.97	5.60	10.7	21.7
$\overline{E}_\pi$		1.17	1.27	1.75	2.56	3.72	5.04	7.17	14.0	28.8
$\overline{E}_0$		4.19	5.44	7.90	12.6	18.9	25.8	36.6	67.7	12.6
m		1.58	1.89	2.17	2.81	3.25	3.63	4.48	7.20	10.67
Δ		0.21	0.23	0.25	0.28	0.30	0.31	0.32	0.32	0.33
k		0.03	0.04	0.07	0.08	0.09	0.10	0.11	0.13	0.16
N_μ		3.2	4.7	6.5	9.1	10.7	12.5	16.1	32.2	56.9

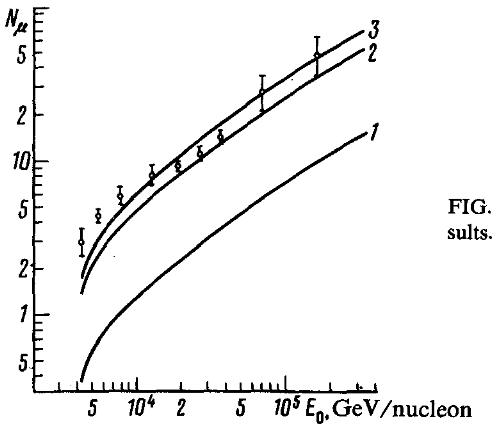

FIG. 2. Comparison of experimental and theoretical results.

muons which are responsible for the cascade and, correspondingly, a decrease in the parameter r_{μ} as a function of the spatial distribution of muons. The quantity Δ depends on the distance from the axis of the group to the axis of the cascade and to the center of the apparatus, the geometric dimensions of the apparatus, and the spatial distribution function of the muons. In the present study, we use the distribution from Ref. 9:

$$F(r, E_{\mu}, \theta) = E_{\mu}^{-0,7} \exp[-(r/r_{\mu})^{0,7}],$$

where $r_{\mu} = 220 r_0 / E_{\mu} \cos\theta$, $r_0 = 6.2$ m. (b) As the cascade energy increases, there is an increase in the "loss" of muons in the part of the apparatus "occupied" by the cascade. The value of c also depends on the spatial distribution of the muons and the dimensions of the region occupied by the cascade. Values of Δ and c are listed in Table I.

4. Figure 2 compares the resulting function $n_{\mu}(E_0)$ with the corresponding function calculated in Refs. 10 and 11 for primary nucleons; the results agree quite well. For this comparison we need to introduce the concept of an average atomic number of the primary nuclei. Knowing the abundance ρ_i of nuclei A_i with a given energy per nucleon, we can find $\rho_{0i} = \rho_i A_i / \Sigma_i \rho_i A_i$, which represents the relative number of nucleons from nucleus A_i with a given energy per nucleon. The averge atomic number is then

$$\bar{A} = \sum_i \rho_{0i} A_i / \sum_i \rho_{0i} = \sum_i \rho_i A_i^2 / \sum_i \rho_i A_i .$$

We have used the chemical composition $\rho_1 = 0.939$, $\rho_4 = 0.055$, $\rho_9 = 0.0009$, $\rho_{14} = 0.0035$, $\rho_{28} = 0.0011$, $\rho_{56} = 0.0003$, from measurements at energies ~ 1 GeV/nucleon and confirmed by measurements at the Baksan telescope through an analysis of the multiplicity spectrum of muon groups.[12] This chemical composition corresponds to $\bar{A} = 3.5$. Curves 1–3 in Fig. 2 correspond to the values $\bar{A} = 1$, 3.5, and 4.5. The data of the present study agree better with $\bar{A} = 4.5$, but this discrepancy with the low-energy data is unimportant because of possible systematic errors. We may conclude that there is no change in the average atomic number of the primary nuclei over the energy range 1–10^5 GeV/nucleon.

[1] A. E. Chudakov *et al.*, Proceedings of the Sixteenth ICRC, Vol. 10, 1979, p. 188.
[2] A.E. Chudakov *et al.*, Proceedings of the Sixteenth ICRC, Vol. 10, 1979, p. 175.
[3] Yu. M. Andreev *et al.*, Izv. Akad. Nauk SSSR, Ser. Fiz. **44**, 613 (1980).
[4] E. V. Bugaev *et al.*, Kosmicheskie myuony i neitrino (Cosmic Muons and Neutrinos), Nauka, Moscow, 1970, p. 15.
[5] S. Hayakawa, Cosmic Ray Physics, Wiley, New York, 1969 (Russ. transl. Mir, Moscow, 1973, Vol. 1, p. 511).
[6] E. V. Bugaev *et al.*, Kosmicheskie myuony i neĭtrino (Cosmic Ions and Neutrinos), Nauka, Moscow, 1970, p. 23.
[7] L. B. Bezrukov and E. V. Bugaev, Proceedings of the Seventeenth ICRC, Vol. 7, 1981, p. 90.
[8] A. M. Hillas, Proceedings of the Sixteenth ICRC, Vol. 6, 1979, p. 12.
[9] V. N. Bakatanov *et al.*, Proceedings of the Eighteenth ICRC, Vol. 11, 1983, p. 453.
[10] I. W. Elbert and P. Summers, Preprint UUHEP 83/20, Univ. of Utah.
[11] A. D. Erlykin *et al.*, Izv. Akad. Nauk SSSR, Ser. Fiz. **46**, 1836 (1982).
[12] E. V. Budko *et al.*, Proceedings of Nineteenth ICRE, 1985, HE 5.1-12.

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