Destruction of a metastable vacuum by massive particles

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The probability is calculated for the quantum decay of a metastable vacuum in (I + I) dimensions induced by a massive boson or fermion, which serves as a center for the formation of a critical nucleus of a lower phase. The presence of the particle leads to an exponential factor for an intensification of the decay. Sufficiently heavy particles annihilate the metastable state essentially instantaneously. During the expansion of the critical nucleus, the particle itself is carried off to infinity in the form of a zero mode coupled to a soliton.

The probability for the decay of a metastable vacuum in (I + I) dimensions has recently been found in closed form for both a purely boson system[1,2] and the case in which there is a fermion field,[3,4] which has a zero mode on a soliton that sends the metastable vacuum (Φ_+) into a lower vacuum (Φ_-). The decay is known[5,7] to result from the formation, through quantum tunneling, of a critical nucleus of the lower phase, of length $2R$, followed by an expansion of this nucleus. During the expansion, the transition energy is converted into the kinetic energy of solitons at the ends of the nucleus. The critical radius is determined by the condition that the advantage which stems from the difference in the energy densities of the phases, $\Phi_\pm$, i.e., $\epsilon = \epsilon(\Phi_+) - \epsilon(\Phi_-)$, over a distance $2R$ must be sufficient for the formation of a pair of solitons, each of mass μ: $R = \mu/\epsilon$.

In this letter we briefly describe the method and results of a calculation of the probability for the decay of a metastable vacuum induced by the presence in the Φ_+ phase of a boson of the field Φ or of a fermion. This probability may also be thought of as the decay width of the particle (the imaginary part of its mass). For fermions we consider the case with a zero mode localized at a soliton.

Our result for the decay widths is expressed in terms of the quantity

$$\Gamma(m) = w_0 \, \frac{\mu}{\epsilon} [\sqrt{\pi} \exp(\xi) erfc(\sqrt{\xi})]^2 \exp\left\{ \frac{2\mu^2}{\epsilon} \arcsin \frac{m}{2\mu} + \frac{m\mu}{\epsilon} \sqrt{1 - \frac{m^2}{4\mu^2}} \right\}, \tag{1}$$

where $erfc(x) = (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$, $\xi = m^3/8\mu\epsilon$, and w_0 is the differential, in length and time, probability for the production of a critical nucleus in the absence of particles. For a purely boson system we would have[1,2]

$$w_0 = \frac{\epsilon}{2\pi} \exp(-\pi\mu^2/\epsilon), \tag{2}$$

and for a system containing fermions the value of w_0 would double for each zero fermion mode.[3,4] For the decay widths of the boson (σ) and the fermion (f) we find

$$\Gamma_\sigma = 4A^2\,\Gamma(m_\sigma), \quad \Gamma_f = F^2\,\Gamma(m_f). \tag{3}$$

The constants A and F are determined by the asymptotic behavior of, respectively, the classical background of the field $\Phi(x)$ and the normalized coordinate function $\Psi(x)$ of the zero mode of a fermion at a soliton with increasing depth into the Φ_+ phase: $(\Phi(x) - \Phi_+) \to Ae^{-mx}$, $\Psi(x) \to F\sqrt{m}e^{-mx}$. The value of A is on the order of $(\Phi_+ - \Phi_-)$, $F \to 1$, when $m_f \ll m_\sigma$.

These expressions for the widths were derived under the condition that the following inequalities hold:

$$m_{f,\sigma}R = (m_{f,\sigma}\mu/\epsilon) \gg 1, \quad m_{f,\sigma} \ll \mu. \tag{4}$$

The exponential factor in (1), however, is exact in terms of the parameter m/μ. We are also making no assumption regarding the magnitude of the parameter ξ. [It is not difficult to see that the expression in brackets in (1) does not contain an exponential function; it has the behavior $1/\sqrt{\xi}$ in the limit $\xi \to \infty$, and it approaches $\sqrt{\pi}$ in the limit $\xi \to 0$.] If m is greater than 2μ, the quantity $\Gamma(m)$ has no small exponential factor, so that a metastable state of a particle is destroyed instantaneously. The same effect can be expected in the case $m < 2\mu$ if a multiparticle boson state with a mass $M > 2\mu$ and a size $r \ll R$ is produced. (This could not happen for fermions, since the zero mode can absorb only a single fermion.)

The tunneling trajectory is described[8] by the field configuration in a Euclidean space-time, which, in the absence of particles, is a circular "bubble" of radius R filled with the Φ_- state (outside is the Φ_+ state). The circular bubble is a stationary configuration for the action[2]

$$S = \mu L - \epsilon A, \tag{5}$$

where L is the perimeter, and A the area, of the bubble. Let us examine the evolution of the wave function of the particle in Euclidean time. In the absence of a bubble, the wave function acquires a factor $\exp(-mT)$ in a time T $(T \gg R)$ (Fig. 1a). If, however, there is a bubble on the world line of the particle, the presence of soft modes localized at the boundary of the bubble[1,4] will cause the particle to propagate along the surface of the bubble as a massless particle, so that the exponential evolutionary factor takes the form $\exp[-m(T-H)]$, where H is the bubble diameter in the time direction. Accordingly, taking the single-bubble contribution into account, we find that the be-

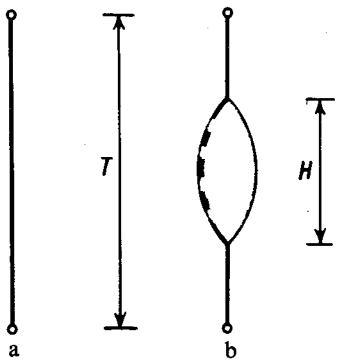

FIG. 1. Semiclassical trajectories of a particle (heavy lines) and of solitons (light line) in Euclidean space. The heavy dashed line shows the propagation of a particle in the form of a soft mode.

havior of the wave function is $\exp(-mT)\times(1-CT\exp(mH))$. Comparing this behavior with an expansion of the expected exact result, $\exp(-(m+\delta m)T)$, we find $\delta m = C\exp(mH)$. To find the coefficient of the exponential function, CT, we need to evaluate the functional integral over fluctuations around a configuration with a single bubble (the factor T obviously arises from the integration over the position of the bubble in time). It should be taken into account that the equilibrium (classical) shape of the bubble is distorted by the term mH in the exponential function, which should be subtracted from action (5): $S_m = S - mH$. A stationary configuration for S_m is a spindle-shaped bubble (Fig. 1b). The angle at the vertex of the spindle is $2\arccos(m/2\mu)$, and the sides are two circular arcs of radius $R=\mu/\epsilon$. The action S_m on this configuration gives us the argument of the exponential function in (1)–(3). At $m=2\mu$, the bubble contracts to a point, and the process ceases to be semiclassical.

The imaginary part of the coefficient of the exponential function, which determines the decay probability, is calculated in the standard way.[1–4] In the Green's function we retain only the soft-mode contribution [since only these modes provide a factor of $\exp(mH)$]; for functions of these modes under the condition $mR \gg 1$, the only important considerations are their exponential asymptotes with distance from the bubble wall. We also note that our exponential function in the expression for Γ_σ agrees in the case $m \ll \mu$ with that derived in Ref. 9 by a classical approach. A fundamentally new point is the assertion of a significant distortion of the stationary shape of the bubble.

We note in conclusion that the structure of the final state, which is determined by the field configuration in the equatorial cross section of the bubble (Fig. 1b), corresponds to a soliton and a soliton filled by a zero mode. During expansion in Minkowski space, these solitons go off to infinity, carrying the particle with them. Incidentally, for a boson the zero mode is simply a displacement of a soliton (in other words, the presence of a boson alters the initial conditions for the classical expansion of the nucleus). The boson thus "dissolves" in the vacuum.

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