Variation of the random potential and the conductivity of samples of small dimensions

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(Submitted 12 September 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **42**, No. 9, 363–365 (10 November 1985)

The residual conductivity of samples of small dimensions is highly sensitive to small changes in the impurity potential. The conductivity of a film of finite thickness changes, for example, when an impurity is moved a finite distance, regardless of the film dimensions. This effect can be used to study extremely slow diffusion of impurities. The magnetoresistance of a spin glass, produced as a result of changing the way in which the exchange potential is produced in a magnetic field, is considered. A possible link between this effect and the experiments carried out by Webb *et al.*[5] [Phys. Rev. B **30**, 4048 (1984); Phys. Rev. Lett. **51**, 2696 (1985)] is discussed.

1. The conductivity of small-diameter conductors fluctuates from one sample to the next. These fluctuations were analyzed by Al'tshuler[1] and Lee and Stone[2] under conditions of weak localization. At $T=0$ the deviation of the conductance of the sample from the mean value $\langle G \rangle$ was found to be on the order of $e^2/\hbar$ (irrespective of its size or shape). The presence of these fluctuations makes the electrical conductivity extremely sensitive to small changes in the random potential which scatters the conduction electrons. Suppose that this potential for some reason changes from $u(\mathbf{r})$ to $u(\mathbf{r}')$. We will show that the correlation function F of the conductances before the change (G) and after the change (G') is

$$F \equiv \langle GG' \rangle - \langle G \rangle \langle G' \rangle \doteq \left(\frac{2e^2}{\pi\hbar}\right)^2 \times \begin{cases} \dfrac{d+1}{\pi^4} b_1 - \dfrac{4b_2}{\pi^2}\left(\dfrac{L}{L_f}\right)^2 & L < L_f, \\ \dfrac{\Gamma\left(\frac{d}{2}\right)}{2\pi^{d/2}}\left(\dfrac{L_f}{L}\right)^{4-d} & L > L_f \end{cases} , \tag{1}$$

where

$$L_f = \sqrt{D\tau_f} = l/\sqrt{\alpha}; \alpha = 1 - \frac{\langle UU' \rangle}{\langle U^2 \rangle} . \tag{2}$$

Here b_1 and b_2 are coefficients which depend on the shape of the sample, d is the effective dimensionality with respect to the length L_f, L is the dimension of the sample in the direction of the current, l is the mean free path, and D is the diffusion coefficient of electrons.

2. If the conductivity is determined by the scattering by a short-range impurity potential, then $\alpha \cong \delta N_{im}/N_{im}$, where $N_{im} = VC_{im}$ is the total number of impurities in the sample, and δN_{im} is the number of impurities that have changed their position. We see that for $\delta N_{im} \sim l^2 a C_{im}$ (C_{im} is the impurity concentration) the condition $D\tau_f \sim L^2$ (which leads to $|G - G'| \sim e^2/\hbar$) is satisfied in a square film of finite thickness a for any value of L no matter how large. In other words, at $T = 0$ and $d = 2$ a change in the position of *only* the impurity changes the conductivity by a *finite value*. In general, the conductivity changes by a finite amount when δN_{im} is proportional to L^{d-2}.

The high sensitivity of the conductance to a change in the position of the impurities makes it possible in principle to study the diffusion of impurities in metals (the quantum diffusion of hydrogen, for example) at very low diffusion coefficients D_{im}. If the impurity jumps a distance r_0 over a scale time T_{im}, the conductance will change by the amount $e^2/\hbar$ in a time $t \sim T_{im}(l^2/L^2)$. This allows one to measure $D_{im} \sim r_0^2 l^2/L^2 t$ in a time t. With $r_0^2 \sim 10^{-15}$ cm^2, $l \sim 10^{-6}$ cm, and $L \sim 10^{-4}$ cm it is possible to measure $D_{im} \sim 10^{-19}$ cm^2/s in a time $t \sim 1$ s.

3. If the sample has localized spins, then at low temperatures these spins will either form a spin glass at $T_G > T$, T_K or vanish due to the Kondo effect ($T_K > T_G$, T).[1)] Here T_K and T_G are the Kondo temperature and the temperature for the transition to the spin glass, respectively. In either case, the magnetic field H has an effect on the scattering potential which is caused by the exchange interaction of localized spins with the conduction electron spins (we assume that no actual localized spin flips occur as a result of scattering of conduction electrons by these spins).

The value L_f in the spin glass arising from this change has the form, which is analogous to (2),

$$L_f = \sqrt{\frac{D\tau_s}{\alpha_{SG}(H)}} \quad ; \quad \alpha_{SG}(H) = 1 - \frac{\langle S(0)S(H) \rangle}{S(S+1)} , \tag{2a}$$

where S is the localized spin, and τ_s is the free flight time of an electron relative to the scattering by a localized spin. Substituting (2a) into (1), we find that $F(H) = \langle G(H)G(0)\rangle - \langle G(H)\rangle\langle G(0)\rangle$ decreases substantially with increasing H at the level of H_s which is determined from the condition $\alpha_{SG}(H_s) \sim D\tau_s/\hbar^2$. This means that the conductance of a single sample oscillates randomly as a function of H with a characteristic period H_s and an amplitude on the order of $e^2/\hbar$. The functional dependence $F(H)$, which is associated with the orbital effects, was studied in Refs. 2–4 in order to explain the experiments of Ref. 5. The oscillation amplitude turned out to be the same (on the order of $e^2/\hbar$), while the characteristic period was determined from the condition $H_c L^2 \gamma \sim \Phi_0 = \hbar c/e$. Here $\gamma = A/L^2$, and A is the cross-sectional area of the sample, whose plane is perpendicular to H. Thus, if $\alpha_{SG} \sim (g\mu H/T_G)^\beta$, then

$$\frac{H_s}{H_c} \cong \left(\frac{D\tau_s}{L^2}\right)^{1/\beta - 1} \gamma(T_G\tau_s) \frac{P_F l}{\hbar}, \tag{3}$$

where p_F is the Fermi momentum, and $g\mu H$ is the Zeeman splitting. From (3) we find that any relationship between H_s and H_c is possible, and if the samples are sufficiently dirty, $H_s < H_c$; i.e., the spin effects determine the functional dependence $F(H)$ and hence the irregular oscillations of $G(H)$. From an analysis of these oscillations we can experimentally determine the quantity $\alpha_{SG}(H)$, which is important in the physics of spin glasses and which is very difficult to measure by other methods.

In the experiments of Ref. 5, the authors observed random oscillations of $G(H)$, which can in principle be explained in terms of both the orbital and spin effects. However, the difference between $G(H)$ and $G(-H)$ observed in Ref. 5 cannot, by virtue of the Onsager relations, be explained in terms of the orbital effects, and it should be attributed entirely to the spins. We wish to emphasize that even if $H_c < H_s$, in which case the functional dependence $G(H)$ is determined primarily by the orbital effects, the quantity $G(H) - G(-H)$ is on the order of $e^2/\hbar$ and varies only due to the spin effects at the level H_s. The orbital effects can also be separated from the spin effects by analyzing the time evolution of $\alpha_{SG}(H, t)$ characteristic of spin glass and by studying the dependence of the effect on the magnetic field direction, i.e., the quantity γ. In Kondo's case, we have $\tau_f \sim \tau_s = 4T_K^2/(g\mu H)^2$. This situation also causes random oscillations of $G(H)$, but the relation $G(H) = G(-H)$ remains in force.

4. Al'tshuler[1] and Lee and Stone[2] showed that the sum of the diagrams like those in Fig. 1 must be calculated before the correlation function of the conductivities can be calculated. Equation (1) can be obtained by substituting the impurity potential $u(\mathbf{r})$ into one electron loop in Fig. 1 and the impurity potential $u'(\mathbf{r})$ into the other. Consequently, $\langle u^2\rangle$ or $\langle u'^2\rangle$ appears in the electron propagators and the dashed line, which

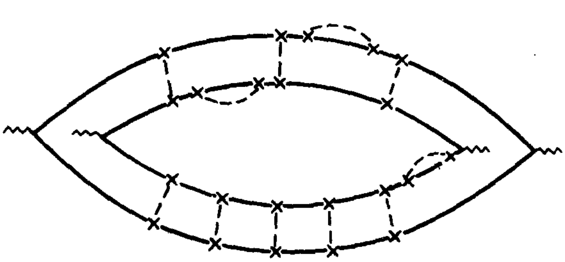

FIG. 1.

connects different loops, corresponds to $\langle uu' \rangle$. As a result, the sum of the ladder diagrams (diffusion or Cooper poles) is equal to $(Dq^2 + \tau_f^{-1})^{-1}$, which leads to (1) if the same boundary conditions as those in Refs. 1 and 2 are used.

5. Another experimental case in which the fluctuation properties of small samples are seen is a superconductor–normal metal–superconductor junction with $L > \sqrt{D\hbar/T}$, in which the Josephson effect is negligible. Spivak and Khmel'nitskiĭ[6] have shown that $\langle G \rangle$ in such a system oscillates as a function of the difference in phases φ of the superconductors with a period π. On the other hand, both the experimental study[5] and the theoretical studies[2–4,6–8] dealing with the Aharonov-Bohm effect in small samples found that the unaveraged conductance oscillates with a "normal" (nonsuperconducting) period Φ_0. For the same reasons, $F(\varphi) = \langle G(\varphi)G(0) \rangle - \langle G(\varphi) \rangle \langle G(0) \rangle$ in the system specified above. Accordingly, $G(\varphi)$ is also a periodic function φ with a period 2π.

We wish to thank A. G. Aronov, D. E. Khmel'nitskiĭ, and B. I. Shklovskiĭ for useful discussions.

[1)]In the paramagnetic region the fluctuations are suppressed by spin scattering at $D\tau_s < L^2$. The discussion of this question in Ref. 1 contains an error.

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Translated by S. J. Amoretty