Optical bistability in a nonlinear system with distributed feedback (experiment)

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Hysteresis has been observed experimentally for the first time in the nonlinear reflection (or transmission) during the illumination of a layer of a cholesteric liquid crystal by a low-power (milliwatt) laser beam. An oscillatory (in time) transition to a steady state has been found.

1. Resonator-free arrangements with optical bistability have recently attracted the greatest interest (Ref. 1, for example). It has been suggested that systems with distributed feedback, which arise either during the dynamic self-diffraction of light waves or in media with a spatially inhomogeneous structure, be used for this purpose (see Ref. 2 and the bibliography there). In addition to the purely physical interest in these problems, they are pertinent to the development of optical logic elements, of great practical importance.[3]

In the present experiments we have, for the first time, arranged a nonlinear reflection (or transmission) of light, resulting in an optical bistability, in a mixture of cholesteric liquid crystals exhibiting a region of selective (Bragg) reflection.[4] The light intensities required here are low, ~ 10 W/cm^2, determined by a thermal nonlinearity mechanism, in constrast with the orientational nonlinearity of cholesteric liquid crystals which was studied in Ref. 5 and which requires far stronger fields ($\gtrsim 1$ MW/cm^2).

2. We use a planar-oriented mixture of cholesteric liquid crystals (cholesteryl pelargonate and cholesteryl oleate) and the nematic liquid crystal 5CB with the respective weight ratios 75:15:10, with a thickness $d = 10$ μm. The pump light is a circularly polarized focused beam from a He-Ne laser ($\lambda = 0.633$ μm), with a maximum intensity $I_{inc} = 60$ W/cm^2 and a diameter of 85 μm.

A dye (methyl blue) is added to the cell to create the required nonlinearity. This nonlinearity arises from the absorption of the laser beam and leads to a change in the Bragg conditions. The absorption coefficient of the mixture is $\beta \cong 990$ cm^{-1}.

Figure 1 shows some typical results on the temperature dependence of the light intensity transmitted (or reflected) by the sample. The temperature is regulated within 0.1 °C.

Figure 2 shows the I_{inc} dependence of T^+ and R^+ as I_{inc} is gradually increased and reduced. There is an obvious hysteresis, which determines the optical bistability in the system. This hysteresis corresponds to an initial tuning to the reflection maximum (point A in Fig. 1).

If the initial tuning corresponds to a point E to the left of point A in Fig. 1, we observe oscillations over time (Fig. 3), with an amplitude that decreases for several

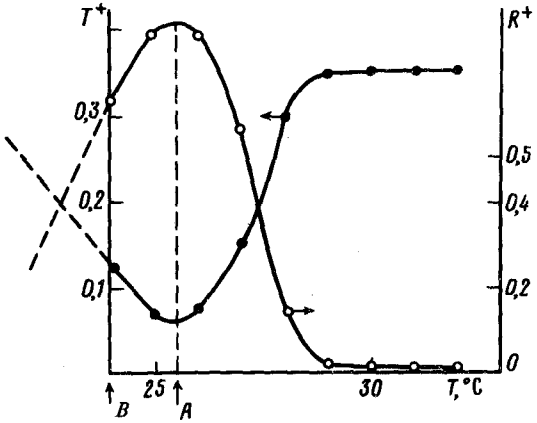

FIG. 1. The transmission $T^+ = I_{tr}/I_{inc}$ and the reflection $R^+ = I_{ref}/I_{inc}$ for the mixture used in these experiments versus the Celsius temperature T. (The left sides of the curves must correspond to a cooling of the sample below room temperature and were not recorded in this case.) Because of absorption, the condition $R^+ + T^+ < 1$ holds; scattering is ignored.

tens of minutes. There is a pronounced asymmetry of the ascending and descending branches of the curves: The scale time for the switching on is far shorter.

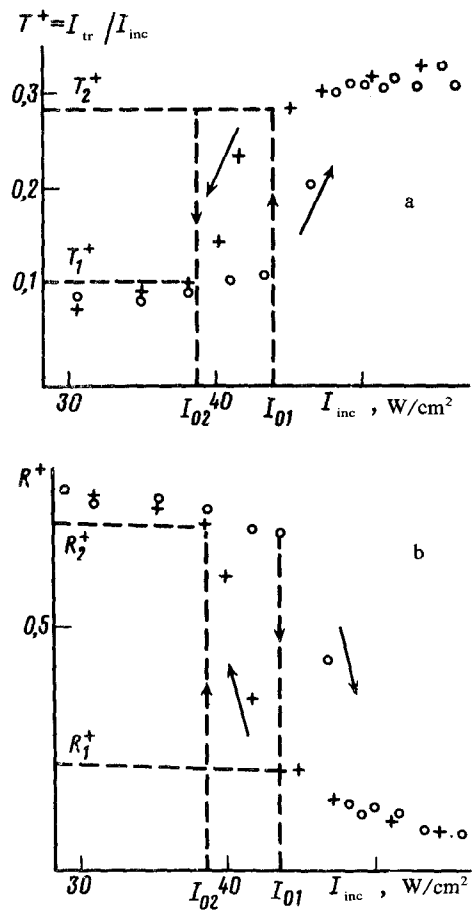

FIG. 2. Hysteresis in the plots of T^+ (a) and R^+ (b) versus I_{inc} as this intensity is gradually increased and reduced (the arrows show the directions). The measurements were taken at steady-state points, after an equilibrium thermal regime had been established for each value of I_{inc}.

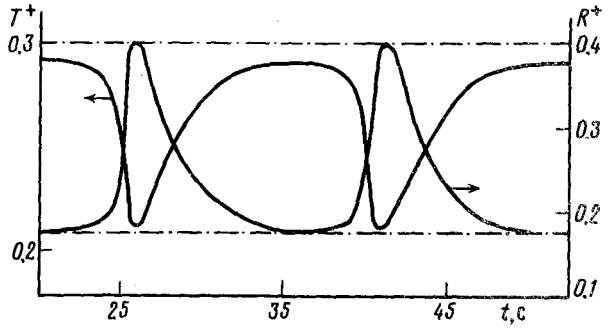

FIG. 3. Typical oscillations of T^+ and R^+ over time which occur just after an "instantaneous" increase in the light intensity.

Other regimes also arise, depending on the initial tuning; in particular, there is a sequence of oscillations with very different amplitudes and time scales.

3. A quantitative analysis of the problem involves a simultaneous solution of the equations for R^+ (or T^+), for the T dependence of the pitch p of the helix of the working mixture, and for the thermal conductivity of the mixture during the laser heating.

The expression for R^+ can be written in the form[1)] (Ref. 4)

$$R^+ = [1 + (c^2/4\omega^2 \bar{\epsilon} \epsilon_a^2) p^2 / \sin^2 k^- d]^{-1}, \tag{1}$$

where $k^- = (\omega \bar{\epsilon}^{1/2}/c)[1 + \mathscr{H}^2 \pm (4\mathscr{H}^2 + \epsilon_a^2)^{1/2}]^{1/2}$, $\mathscr{H} = cp/4\omega\bar{\epsilon}^{1/2}$, ω is the frequency of the liquid, c is the velocity of light, and $\bar{\epsilon}$ is the average value of the dielectric constant of the medium. [In the region of selective reflection, where k^- is imaginary, we need to make the replacement $\sin k^- d \to \sinh |k^-| d$ in (1).]

The temperature dependence of p is given in our case by[4]

$$p = \gamma [1 + \mu (T - T_c)^{-1}]^2, \tag{2}$$

where γ and μ are constants, and T_c is the temperature of the phase transition to an isotropic liquid.

Expressions (1) and (2) determine the T dependence of R^+; we also found this dependence in our experiments (Fig. 1).

We write the heat-conduction equation as[2)] (Ref. 6)

$$\frac{\partial}{\partial T} \delta T = D_T \frac{\partial^2 \delta T}{\partial z^2} + \frac{\beta P_0}{\rho C_p} \frac{2}{\pi r_0^2} \exp(-2r^2/r_0^2) e^{-\alpha z} H(t), \tag{3}$$

where δT is the change in temperature due to the laser heating, D_T is the thermal diffusion coefficient, β is the absorption coefficient, ρ is the density, C_p is the specific heat at constant pressure, $\alpha = \alpha(I_{\text{in}})$ is the overall loss coefficient (which also incorporates the selective reflection and thus depends on the intensity in the medium, I_{in}), P_0 is the total light power for a Gaussian beam, and $H(t)$ is the unit step function, which determines the time at which the pump is turned on.

Integrating (3) over z for the steady-state case, we find

$$\delta T = (1/D_T)(\beta/\alpha^2 \rho C_p) I_{\text{inc}} e^{-\alpha z}, \tag{4}$$

where $I_{inc} = P_0(2/\pi r_0^2)\exp(-2r^2/r_0^2)$. We may assume, for simplicity, that under Bragg-reflection conditions we have $\alpha_{Br}^{-1} \sim p \approx \lambda / \bar{\varepsilon}^{1/2}$; while outside this region we have $\alpha \approx \beta$.

The heating δT caused by the laser beam causes a change in p: $\delta p = (\partial p/\partial T)\delta T$. A change $\delta R^+ = (\partial R^+/\partial p)\delta p$ then arises. System (1)–(4) can thus be used for a comprehensive analysis of the problem.[3)] However, we restrict the discussion here to the qualitative picture.

The physical reason for the optical bistability in our case is the difference in the values of I_{in} for regimes with high transmission and high reflection.[5] For the conditions in Fig. 2, an increase in I_{inc} is accompanied by a transition to a regime with transmission at that value ($I_{inc} = I_{01}$) at which I_{in} (which amounts to only a small fraction of $I_{inc} - I_{in} = I_{inc}e^{-\alpha_{Br}z}$, $\alpha_{Br}z \gg 1$ is sufficient for a change in p. If, on the other hand, we now begin to reduce I_{inc}, since essentially all the light is transmitted through the medium ($I_{in} \approx I_{inc}e^{-\beta z}$), the transition back to the regime with reflection occurs at values $I_{inc} = I_{02} < I_{01}$. The width of the hysteresis curve is determined by the difference between I_{01} and I_{02}, and the size of the "jump" between these two states (I_2/I_1) is determined by the shape of the resonant curve in Fig. 1 and by the dependence $\delta T(I_{inc})$. For Fig. 2 we have $I_{01} - I_{02} \approx 5$ W/cm^2, and I_2/I_1 is on the order of 2.5 for T^+ and 3.5 for R^+.

Let us estimate the value of δT on the basis of (4) and compare the result with that found experimentally. Under the conditions of Fig. 2b we have $I_{inc} \approx 40$ W/cm^2, $\beta \approx 10^3$ cm^{-1}, $\alpha \sim \bar{\varepsilon}^{1/2}/\lambda \approx 2 \times 10^4$ cm^{-1}, and $e^{-\alpha z} \sim 1$; adopting the values $D_T \sim 10^{-4}$ cm^2/s and $\rho C_p \approx 1$ J/(cm^3 · deg), which are typical of cholesteric liquid crystals,[5] we find $\delta T \sim 1$ °C. This result agrees in order of magnitude with the value of δT which can be estimated from Figs. 1 and 2b.

Similar arguments for an initial point of type B in Fig. 1 lead to oscillations over time: The heating of the medium intensifies the reflection; the light does not penetrate into the sample; and the sample cools down. In other words, it reverts to the regime of a higher transmission and thus to a repeated heating. These cycles continue until an equilibrium temperature is established in the sample.[4)] The reasons for the asymmetry of the ascending and descending branches of the oscillations are the difference in temperature gradients during cooling and heating and the different values of the derivatives for the curve in Fig. 1 near points A and B. In our case, these time scales for the changes in p are on the order of 1 and 5 s, respectively; they determine the relaxation process over many periods of the helix.

Clearly, if the system goes through a point A during the heating, there may be, in particular, a sequence of oscillations differing in duration.

The I_{in} dependence enters (1) through p, but expression (1) is similar in form to the expression for R^+ in a nonlinear Fabry-Perot resonator,[1] a classical system with optical bistability. These problems are therefore analogous (it is convenient to analyze them graphically[5)]).

4. Optical bistability can also be observed in our case at light power levels one or two orders of magnitude lower, if β is increased.[6)] Another way to reduce I_{inc} is to use cholesteric liquid crystals with an extremely narrow temperature interval of selective

reflection.[4] If we were to use a reflecting mirror (and a polarizer) to create an additional feedback (cf. Ref. 2), we could reduce I_{inc} even further, and the hysteresis loop would become broader. Another possibility is to use an external heater in contact with the liquid crystal[7]; such a system, however, would be difficult to control. The most interesting possibility is that of inserting the cholesteric liquid crystal into a laser cavity.[1]

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[1)]If the medium has a slight anisotropy, we would have $\epsilon_a \ll 1$ even if we ignored reflection at the boundaries.
[2)]We are considering the one-dimensional problem (along z; the radius of the laser beam satisfies $r_0 \gg 2d$.)
[3)]For the transient regime, it would essentially be necessary to solve the problem of the selective transmission of light through a cholesteric liquid crystal, in which the pitch of the helix varies monotonically along the direction of the light beam.
[4)]When another nonlinearity mechanism is also operating (e.g., an orientational mechanism), and this mechanism has a different sign and a different relaxation time, there may be oscillations which do not decay over time (Ref. 1; cf. Ref. 8).
[5)]The functional dependence $R^+(I_{in})$ in (1) is more complex, and the relationship between I_{ref} and I_{in} becomes dependent on the intensity.
[6)]An increase in β is not very important for I_{tr} for thin layers of cholesteric liquid crystals. The heating can also be intensified by using a linear polarization of the incident light.

[1]S. M. Arakelyan and Yu. S. Chilingaryan, Nelineĭnaya optika zhidkikh kristallov (Nonlinear Optics of Liquid Crystals), Nauka, Moscow, 1984, p. 360.
[2]A. E. Kaplan and C. T. Law, IEEE J. Quantum Electron. No. 9, 1985 (Special issue on Optical Bistability).
[3]Conference on Laser and Electrooptics (CLEO 85), Abstracts, Baltimore, USA, May, 1985.
[4]V. A. Belyakov and A. S. Sonin, Optika kholestericheskikh zhidkikh kristallov (Optics of Cholesteric Liquid Crystals), Nauka, Moscow, 1982, p. 360.
[5]H. G. Winful, Phys. Rev. Lett. **49**, 1179 (1982).
[6]G. Koren, Phys. Rev. A **13**, 1177 (1976).
[7]B. Ya. Zel'dovich and N. V. Tabiryan, Kvantovaya Elektron. (Moscow) **11**, 2419 (1984) [Sov. J. Quantum Electron. **14**, 1599 (1984)].
[8]V. L. Golo and E. I. Kats, Zh. Eksp. Teor. Fiz. **87**, 1700 (1984) [Sov. Phys. JETP **60**, 977 (1984)].

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