# Systematic features of the spin dynamics of $^3$He-$A$ near the transition to the $A_1$ phase

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The spin dynamics of $^3$He-$A$ near the transition to the $A_1$ phase is studied. The criterion for the stability of uniform magnetization precession is determined and the mixing of the entropy oscillations with the transverse spin excitations against the background of precession dynamics in the transition region $A \rightarrow A_1$ is discussed.

The $A_1$ phase which directly follows the normal state and which is characterized by Cooper pairing in a single configuration along the spin projection is known to be stabilized in a superfluid $^3$He in a magnetic field. The capabilities of experimental study of the properties of the $A_1$ phase of liquid $^3$He have recently increased considerably.[1,2] Accordingly, the identification of new effects that occur near the phase transition $A \rightarrow A_1$ has become a subject of current interest. Of particular interest is to theoretically study further the systematic features of the spin dynamics of $^3$He-$A$ near the transition to the $A_1$ phase during the imposition of pulses, which causes the magnetization to deflect by a certain angle $\theta$ from the equilibrium orientation and to precess around the external magnetic field at a frequency $\omega_{\perp}$.

In a strong field, the shift of the precession frequency relative to the Larmor

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precession, $\omega_0 = \gamma H_0$, is given by[3]

$$\delta\omega_\perp = \frac{1}{8}\frac{\Omega_A^2}{\omega_0}\{1 + [2+\beta - (6+\beta)\sin^2\delta]\cos\theta\}, \tag{1}$$

where $\Omega_A$ is a characteristic dipole frequency, $\delta$ is the equilibrium angle between the spin axis $\mathbf{d}_1$ and the fixed orbital axis $\mathbf{l}$, and the parameter $\beta = \Delta_\uparrow\Delta_\downarrow/\Delta^2$ is a measure of the proximity to the $A_1$ phase [here $\Delta_\uparrow\Delta_\downarrow$ represents the pairing amplitude in the spin configuration $\uparrow\uparrow$ $(\downarrow\downarrow)$, where $\Delta^2 = \frac{1}{2}(\Delta_\uparrow^2 + \Delta_\downarrow^2)$]. In this letter we will determine the criterion for the stability of a uniform magnetization precession relative to the long-wave spin excitations and we will study the mixing of the oscillatory entropy (and temperature) with these excitations near the transition $A \to A_1$.

The order parameter of $^3$He-$A$, which is placed in an external magnetic field, is

$$A_{\mu i} = \Delta d_\mu u_i, \tag{2}$$

where the spin vector is $\mathbf{d} = \alpha_+\mathbf{d}_1 + i\alpha_-\mathbf{d}_2$, $\mathbf{d}_1 \perp \mathbf{d}_2$, and the coefficient $\alpha_\pm = \frac{1}{2\Delta}(\Delta_\uparrow \pm \Delta_\downarrow)$. As for the orbital vector $\mathbf{u}$ when $\mathbf{l}$ is fixed, it depends solely on the local phase, whose gradient determines the superfluid velocity $\mathbf{v}_s$.

The Galilean-invariant part of the inhomogeneity energy is

$$F_g = \frac{1}{2}\rho_{ij}^{(s)}(w_{si}w_{sj} + \alpha_-^2 w_{1i}w_{1j} + \alpha_+^2 w_{2i}w_{2j} + w_{3i}w_{3j} + 4\alpha_-\alpha_+ w_{si}w_{3j}), \tag{3}$$

where $\mathbf{w}_s = \mathbf{v}_s - \mathbf{v}_n$, and the superfluid spin velocities are given $(\mathbf{d}_3 = \mathbf{d}_1 \times \mathbf{d}_2)$

$$\mathbf{w}_1 = \frac{\hbar}{2m} d_{2\mu}\vec{\nabla} d_{3\mu},\quad \mathbf{w}_2 = \frac{\hbar}{2m} d_{3\mu}\vec{\nabla} d_{1\mu},\quad \mathbf{w}_3 = \frac{\hbar}{2m} d_{1\mu}\vec{\nabla} d_{2\mu}. \tag{4}$$

Expressing the spin velocities in terms of Euler's angles $(\varphi,\theta,\psi)$ and averaging them over the "fast" variables, we find

$$\bar{F}_g = \frac{1}{2}\rho_{ij}^{(s)}\Big[w_{si}w_{sj} + \left(\frac{\hbar}{2m}\right)^2\Big(\nabla_i\phi\nabla_j\phi + \frac{1}{2}\nabla_i\theta\nabla_j\theta + \frac{1}{2}(1-\cos\theta)(3-\cos\theta)\nabla_i\varphi\nabla_j\varphi$$

$$- 2(1-\cos\theta)\nabla_i\varphi\nabla_j\phi\Big) + 2\alpha\frac{\hbar}{2m} w_{si}(-\nabla_j\phi + (1-\cos\theta)\nabla_j\varphi)\Big], \tag{5}$$

where $\phi = \varphi + \psi$ is the "slow" variable, and $\alpha = 2\alpha_+\alpha_- = (\Delta_\uparrow^2 - \Delta_\downarrow^2)/(\Delta_{\uparrow\uparrow}^2 + \Delta_\downarrow^2)$.

On the other hand, the average dipole-dipole energy of the superfluid $^3$He-$A$ has the form

$$V_D = -\frac{1}{8}\chi(\Omega_A/\gamma)^2[(1-3\sin^2\delta)(1+P/M)^2 + \frac{1}{2}\beta\cos^2\delta(2+P/M)^2\cos 2\phi], \tag{6}$$

where $\chi$ is the magnetic susceptibility, $M$ is the magnetization, and $P = M_z - M = -(1-\cos\theta)M$.

Transforming to dimensionless variables $\xi = M/M_s$ and $\eta = P/M_s$, where $M_s = (\hbar\gamma/2m)\rho$ is the saturation magnetization, we can easily construct the equation of continuity

$$M_s \dot{\xi} + \vec{\nabla} \mathbf{j}_M = -\gamma (\partial V_D / \partial \phi),$$
$$M_s \dot{\eta} + \vec{\nabla} \mathbf{j}_p = 0 . \quad (7)$$

The corresponding densities of the nonconvective components of the flux can be calculated directly by using the procedure of Ref. 5.

Using Hamilton's equations for $\varphi$ and $\phi$ (Ref. 4) and also the equations for the oscillatory entropy $\delta\sigma$ (Ref. 5), we find the following system of equations which describes the collective excitations in $^3$He-$A$ against the background of a precessional motion of magnetization:

$$(\omega^2 - \omega_\parallel^2(k))\delta\xi + \alpha\omega_2^2(k)\delta\sigma/\sigma - \frac{2\epsilon}{1+\cos\theta}\Omega_\parallel^2\delta\eta = 0,$$

$$(\omega^2 - \omega_2^2(k))\delta\sigma/\sigma + \alpha\omega_{sp}^2(k)\delta\xi = 0, \quad (8)$$

$$(\omega^2 - \epsilon\bar{\omega}_{sp}^2(k))\delta\eta + (1-\cos\theta)[\omega_{sp}^2(k)\delta\xi - \alpha\omega_2^2(k)\delta\sigma/\sigma] = 0;$$

here $\omega_\parallel^2(k) = \Omega_\parallel^2 + \omega_{sp}^2(k)$, $\omega_{sp}(k) = u_{sp}k, \omega_2(k) = u_2 k, \bar{\omega}_{sp}^2 = (\rho_n/\rho)\omega_{sp}^2$, $\Omega_\parallel^2 = \frac{1}{4}(1+\cos\theta)^2\beta\Omega_A^2\cos^2\delta$, is the square of the frequency of the longitudinal resonance, $u_{sp}$ and $u_2$ are, respectively, the velocities of the spin wave and of the second sound in $^3$He-$A$ far from the $A_1$ phase, and the small parameter is

$$\epsilon(k) = \frac{1}{4}\left\{ -\frac{1}{4}\sin^2\theta\,(\Omega_A/\omega_0)^2\,[2+\beta-(6+\beta)\sin^2\delta] + \omega_{sp}^2(k)/\omega_0^2 \right\} \quad (9)$$

[we took into account the fact that $(\xi,\eta) \ll 1$].

In the zeroth approximation in $\epsilon$, the system of equations in Ref. 8 describes two branches of collective hybridized oscillations, $\xi$ and $\sigma$, near the transition to the $A_1$ phase.[5] The third branch with

$$\omega_p^2(k) = \epsilon(k)\left[\bar{\omega}_{sp}^2(k) + 2\,\frac{1-\cos\theta}{1+\cos\theta}\,\frac{\beta^2\Omega_\parallel^2}{\Omega_\parallel^2 + \beta^2\omega_{sp}^2(k)}\,\omega_{sp}^2(k)\right] \quad (10)$$

corresponds to the collective transverse excitations against the background of a uniform magnetization precession. At $\epsilon(0) < 0$ the uniform precession is unstable against the decay into long-wave spin excitations. This situation in fact occurs in the case of Leggett's spin-orbit configuration with $\delta = 0$, as was predicted by Fomin[4] and confirmed experimentally by Borovik-Romanov *et al.*[6] Drawing our attention to Eq. (9), we see that the uniform magnetization precession is stable in the region

$$\sin^2\delta > (2+\beta)/(6+\beta), \quad (11)$$

whose width depends on how close it is to the $A_1$ phase. We note that the precession frequency $\omega_\perp = \omega_0 + \delta\omega_\perp$ does not depend on the angle $\theta$ at the boundary of the stability region.

It is easy to see that in the $p$ branch ($\omega = \omega_p$) the oscillations of $\eta$ are accompanied by oscillations of the entropy

$$\delta\sigma/\sigma = \frac{2\epsilon}{1+\cos\theta}\,\frac{\alpha\beta^2\Omega_\parallel^2}{\Omega_\parallel^2+\beta^2\omega_{sp}^2}\,\frac{\omega_{sp}^2}{\omega_p^2-\beta^2\omega_2^2}\,\delta\eta\,. \tag{12}$$

Curiously, the mixing of the oscillations $\delta\sigma$ and $\delta\eta$ occurs only in the transition region $A\to A_1$: The entropy in the $p$ branch does not oscillate far from the $A_1$ phase (where $\alpha=0$) or in the $A_1$ phase (where $\beta=0$).

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