Critical indices in a *XXZ* Heisenberg magnet

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The correlation functions of a Heisenberg *XXZ* magnet at zero temperature are analyzed. The asymptotic behavior of these correlation functions at infinity is studied.

The Hamiltonian of a Heisenberg magnet is

$$H=-\sum_{n=1}^{N}\{\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y+\cos 2\eta(\sigma_n^z\sigma_{n+1}^z-1)+h(\sigma_n^z-1)\}\ . \tag{1}$$

Here σ are the Pauli matrices situated in the periodic lattice with N sites, h is the magnetic field, and η is the coupling constant. Let us consider the following antiferromagnetic region: $0<2\eta\leqslant\pi$. The model was solved by Yang and Yang[1] with the help of the Bethe ansatz. The ground state of the Hamiltonian occurs upon filling of the Dirac sea. The function $\rho(\lambda)$, which describes the distribution of particles in the sea, satisfies the integral equation

$$\rho(\lambda)-\frac{1}{2\pi}\int_{-\Lambda}^{\Lambda}K(\lambda,\mu)\rho(\mu)d\mu=\frac{\sin 2\eta}{2\pi\cosh(\lambda+i\eta)\cosh(\lambda-i\eta)}\ ; \tag{2}$$

$$K(\lambda,\mu)=\sin 4\eta\,[\sinh(\lambda-\mu+2i\eta)\sinh(\lambda-\mu-2i\eta)]^{-1}. \tag{3}$$

The total density D of the sea is

$$D=\int_{-\Lambda}^{\Lambda}\rho(\lambda)d\lambda\ . \tag{4}$$

This density determines the mean value of σ^z in a vacuum: $\langle\sigma_n^z\rangle=1-2D$. In these equations λ is the additive spectral parameter. The bare momentum and bare energy of the particles in the Dirac sea can be described in terms of λ in the following way:

$$p_0(\lambda)=i\ln\frac{\cosh(\lambda-i\eta)}{\cosh(\lambda+i\eta)}\ ;\ \epsilon_0(\lambda)=-2\sin 2\eta\,\frac{dp_0(\lambda)}{d\lambda}+2h\ . \tag{5}$$

Λ is the spectral parameter at the boundary of the Fermi zone. The dressed-charge function $z(\lambda)$ also plays an important role.[2] This function is determined by an integral equation

$$z(\lambda)-\frac{1}{2\pi}\int_{-\Lambda}^{\Lambda}K(\lambda,\mu)z(\mu)d\mu=1. \tag{6}$$

In zero magnetic field ($h\to 0,\Lambda\to\infty$) the charge is $z(\lambda)=\pi/4\eta$ at $|\lambda|\ll\Lambda$; i.e., it reduces to a dressed charge of an elementary excitation.[2] In this model the elementary excitation is viewed as a hole in the Dirac sea and has fermion quantum numbers. The other important characteristic of the model is the velocity of sound v

$$v = \partial\epsilon/\partial p|_{p=\Lambda}. \tag{7}$$

Here $\epsilon(\lambda)$ and $p(\lambda)$ are the dressed (observable) energy and dressed momentum of the elementary excitation.[1] In zero magnetic field we have[3] $v = 2\pi \sin 2\eta/(\pi - 2\eta)$.

We have analyzed the asymptotic behavior of simultaneous correlation functions in a finite magnetic field. We have found the following representation [see Eq. (4)]:

$$\langle \sigma^z_{n+1}\sigma^z_1\rangle - \langle \sigma^z_n\rangle^2 = \frac{a}{n^2} + \frac{b\cos(2\pi Dn)}{n^\theta} \quad (n\to\infty). \tag{8}$$

The critical index θ turned out to be

$$\theta = 2[z(\Lambda)]^2\ ;\quad 0 \leqslant h < 4\sin^2\eta. \tag{9}$$

In the region $0 < 2\eta < \pi/2$ the first term in (8) is the leading term ($\theta > 2$) and in the region $\pi/2 < 2\eta < \pi$ the second term is the leading term ($\theta < 2$). We will explain the reasoning used to derive Eq. (9). The algebraic formulation of the Bethe ansatz plays an important role.[4] In Refs. 5 and 6, all models that have the given R matrix were classified and it was shown that the arbitrary model which is associated with the given R matrix is parametrized by two arbitrary functions $p_0(\lambda)$ and $\epsilon_0(\lambda)$ that replace (5). It follows from Ref. 7 that the critical index θ is independent of these arbitrary functions and is determined by the R matrix and by Λ. This is the key consideration. Equation (9) is consistent with the results of perturbation theory. In the vicinity of free fermions ($2\eta \sim \pi/2$) at $h = 0$ the correlation functions were studied in Refs. 8–10, where it was shown that a single critical index determines the asymptotic behavior of all correlation functions, including the nonsimultaneous correlation functions. On this basis we can advance the following hypothesis for the asymptotic behavior of nonsimultaneous correlation functions:

$$\langle \sigma^-_{n+1}(t)\sigma^+_1(0)\rangle = C(n^2 - v^2t^2)^{-1/2\,\theta} \qquad (n\to\infty).$$

Here v is given by Eq. (7), and θ is given by Eq. (9). Let us now consider the zero-magnetic-field limit ($h\to 0, \Lambda\to\infty$); here $D = 1/2, \langle\sigma^z\rangle = 0$, and $z(\Lambda) = \sqrt{\pi/4\eta}$, so that $\theta = \pi/2\eta$. We note that this equation was used in Ref. 9. The case $2\eta = \pi$ corresponds to an antiferromagnetic case of a Heisenberg XXX magnet. In this case we find the following relations for the simultaneous correlation functions:

$$\langle \sigma^z_{n+1}\sigma^z_1\rangle = b(-1)^n/n\ ;\ \ \langle \sigma^-_{n+1}\sigma^+_1\rangle = C/n \qquad (n\to\infty).$$

We wish to emphasize again that Eq. (9) obtained above for the critical index is a universal equation for any integrable one-dimensional model which has an R matrix of a Heisenberg XXZ or XXX model.[5,6] In all these models (for the given R matrix), the critical index is the same function of Λ (the spectral parameter values at the boundary of the Fermi zone). If, however, the critical index is expressed as a function of the dressed Fermi momentum (as a function of the density), then this index clearly depends on a specific model. In the case of a one-dimensional Bose gas with a point interaction, for example, Eq. (9) {after replacing the kernel K in (6) by a rational kernel: $K(\lambda,\mu) = 2c/[c^2 + (\lambda - \mu)^2]$} reproduces the familiar results obtained in Refs. 11–13.

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