String theories without two-dimensional anomalies

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The conditions for the cancellation of the Weyl and Lorentz anomalies in the two-dimensional chiral supergravities are analyzed. Three anomaly-free theories which have a two-dimensional supersymmetry $N=1/2, N=1_{++}$, and $N=3/2_{++-}$ and which correspond to the string theories with critical dimensions $D=10$ (heterotic string), $D=2$, and $D=2$, respectively, are found to exist.

Attempts to formulate a unified fundamental interaction theory on the basis of the superstring theory has recently attracted considerable interest.[1] The $E_8 \times E_8$ heterotic superstring model seems to have the best chance of developing a realistic low-energy phenomenology.[2] The action of a heterotic string can be represented as the action of $d=2, N=1/2$ supermultiplets that interact with $d=2, N=1/2$ supergravity.[1)] It is therefore important to find all possible consistent string theories which are based on chiral $d=2$ supergravities.

A very simple consistency condition (which appears to be a necessary condition for the existence of massless modes in the ground state of the string) is the cancellation of two-dimensional anomalies in the string action. It was shown in Refs. 5–7 that the critical dimensions (26, 10, and 2) of the three known string models can be determined from the condition for cancellation of the (super) Weyl anomalies in the corresponding $N=0$, 1, and 2 (nonchiral) supersymmetric theories. It was mentioned in Ref. 7 that the theories based on the $N>2, d=2$ supergravities necessarily have a Weyl anomaly. We will therefore restrict the discussion to the chiral $N \leqslant 2, d=2$ supergravities ($N=1/2_+, N=1_{++}, N=3/2_{+++}, N=3/2_{++-}, N=2_{++++}$, and $N=2_{+++-}$) which can be found by "truncation" (and by a chiral "rotation" of some of the fields) from the nonchiral $N=2, d=2$ supergravity[8] which includes a graviton e_m^a, two Majorana gravitinos χ_m^A, and a vector A_m $(a,m,A=1.2)$. The $N=2$ theory is invariant under the Weyl transformations $(\delta e_m^a = \lambda e_m^a)$, standard coordinate transformations (X), local Lorentz transformations (L), "ordinary" supersymmetry ($\delta\chi_m^A = D_m\epsilon^A$), and "conformal" supersymmetry ($\delta\chi_m^A = \gamma_m\zeta^A$) and also under the "phase transformation ($\delta A_m = \partial_m\alpha$) and "axial" transformation ($\delta A_m = \epsilon_{mn}\partial^n\beta$). Assuming that the X invariance is conserved at the quantum level, we can group the anomalies into two multiplets: the super-Weyl anomalies (λ,ζ^A, and β) and the super-Lorentz anomalies (L,ϵ, and α). The super-Lorentz anomalies automatically cancel out in the nonchiral theories, whereas the $D=2$ (Ref. 7) $N=2$ scalar multiplets $(x^\mu, y^\mu, \psi^{\mu A})$ $(\mu = 1,...,D)$ must be added in order to reduce the super-Weyl anomalies to $N=2$ supergravity. We will analyze the conditions under which only the Weyl and Lorentz anomalies are cancelled, which is sufficient for the cancellation of the super-Weyl and super-Lorentz (single-loop) anomalies in the theories that are supersymmetry theories at the classical level.

The finite parts of the effective action for a real scalar and a Majorana spinor in an external gravitational field in $d=2$ are (see Ref. 5, for example)

$$\Gamma_0 = \frac{1}{2}\ln\det\Delta_0 = -\frac{1}{96\pi}\int R\Delta_0^{-1}R, \quad \Delta_0 = -\nabla_m\nabla^m, \tag{1}$$

$$\Gamma_{1/2} = -\frac{1}{2}\ln\det(i\gamma^m D_m) = -\frac{1}{4}\ln\det\Delta_{1/2} = \frac{1}{2}\Gamma_0, \tag{2}$$

$$D_m = \partial_m + i\omega_m\gamma_3, \qquad \Delta_{1/2} = -D^2 + \frac{1}{4}R.$$

The effective actions for a graviton and a Majorana gravitino in $d=2$ are caused by ghosts which correspond to the X and ϵ gauges, $g_{mn} = e^{\rho}\delta_{mn}, \chi_m = \gamma_m\chi$.[5–7]

$$\Gamma_2 = -\frac{1}{2}\ln\det\Delta_1 = -26\Gamma_0, \quad \Delta_{1mn} = -\nabla^2_{mn} - \frac{1}{2}Rg_{mn}, \tag{3}$$

$$\Gamma_{3/2} = 2\ln\det(i\gamma_m^T D_m) = \ln\det\tilde{\Delta}_{1/2} = 11\Gamma_0, \quad \tilde{\Delta}_{1/2} = -D^2 - \frac{1}{4}R. \tag{4}$$

In the case of (anti) self-dual antisymmetric tensor ("chiral scalar") and in the case of a Majorana-Weyl spinor and a gravitino, the effective action has an imaginary part which is specified by the Lorentz gravitational anomaly[9,10]

$$\Gamma_{0\pm} = \frac{1}{2}\ln\det(\partial_m \pm i\epsilon_{mn}\partial^n) = -\frac{1}{192\pi}\int R\Delta_0^{-1}(R \mp i\nabla_m\omega^m),$$

$$\Gamma_{1/2\,\pm} = -\frac{1}{4}\ln\det(i\gamma^m D_m)_\pm = \frac{1}{2}\Gamma_{0\pm}, \quad \gamma_3\psi_\pm = \pm\psi_\pm, \tag{5}$$

$$\Gamma_{3/2\,\pm} = 2\times\frac{1}{2}\ln\det(i\gamma_m^T D_m)_\pm = 11\Gamma_{0\pm}. \tag{6}$$

The coefficients of the real and imaginary parts of Γ (i.e., the coefficients W and L of the Weyl and Lorentz anomalies) in the theory with a single graviton and chiral scalars, spinors, and gravitinos are

$$W=\frac{1}{2}\,(n_{0+}+n_{0-})+\frac{1}{4}\,(n_{1/2+}+n_{1/2-})+\frac{11}{2}\,(n_{3/2+}+n_{3/2-})-26\,, \tag{7}$$

$$L=n_{0+}-n_{0-}+\frac{1}{2}\,(n_{1/2+}-n_{1/2-})-11\,(n_{3/2+}-n_{3/2-}). \tag{8}$$

We note that a chiral scalar can always be replaced by two Majorana-Weyl spinors.

Let us consider the solutions of the equation $W=L=0$, beginning with the case $N=0$ supersymmetry. Selecting $n_{0+}=n_{0^-}=D$ from $L=0$, we find $n_{1/2^+}=n_{1/2^-}=n$, i.e., $W=D+1/2n-26$. At $n=0$ we have an ordinary boson string, whereas $n=2r\neq 0$ corresponds to the Bardakci-Halpern[11] string. In the case of $N=1/2$ supersymmetry, we can consider the interaction of $N=1/2_+$ supergravity (e^a_m,χ_{m^+}) with D $N=1/2_+$ scalar multiplets (x^μ,ψ^μ_-) and with $n_{1/2_+}$ spinor multiplets $(\lambda^I_+,F^I)(F^I$ is the auxiliary field that does not contribute to $\Gamma)$. In this case the equations $W=D+1/4D+1/4n_{1/2^+}+11/2-26=0$ and $L=1/2n_{1/2^+}-1/2D-11=0$ have a unique solution: $D=10$, $n_{1/2^+}=32$, which corresponds to the heterotic string.[2]

The $N=1_{++}$ supergravity includes the following fields: $e^a_m,\chi^A_{m^+}$, $A_{m^+}=\epsilon_{mn}A^n_+$. In addition to a graviton and two gravitinos, a single chiral scalar ghost, which corresponds to the gauge $A_{m^+}=0$ (see Ref. 7), also contributes to the effective action, so that we have $n_{0^-}=-2$. Adding D $N=1_{++}$ scalar multiplets $(x^\mu,y^\mu_-,\psi^{\mu A}_-)$ and several spinor multiplets (i.e., "singlet" spinors of positive chirality), we find $W=D+(1/2)D-1+(1/2)D+(1/4)n_{1/2}+11-26$ and $L=2-D+(1/2)n_{1/2+}-D-22$. It follows, therefore, that at $D=2$ and $n_{1/2^+}=48$ the anomalies cancel out.

In the case $N=3/2_{++-}$, the contribution of supergravity to Γ is the same as the total contribution of $N=1_{++}$ and $N=1/2_-$ supergravities (on condition that the contribution from a graviton is taken into account only once). Adding D scalar multiplets $(x^\mu,y^\mu_-,\psi^{\mu A}_+,\psi^\mu_-)$ and $n_{1/2^+}$ chiral "singlet" spinors, we find that the equations $W=L=0$ have a unique solution: $D=2,n_{1/2^+}=24$. A similar analysis shows that the remaining chiral theories with $N=3/2_{+++}$, $N=2_{++++}$, and $N=2_{+++-}$ have either a Lorentz anomaly or a Weyl anomaly. In the case of $N=2$ theories which interact with D $N=2$ scalar multiplets, for example, we have[7] $W=3(D-2)$. To cancel the Lorentz anomaly, we must introduce additional Majorana-Weyl spinors which account for a positive contribution to W (as a result, $W\neq 0$ even if $D<2$).

Accordingly, the total number of string theories, which are free of two-dimensional anomalies, is: 1) $N=0,D=26$; 2) $N=1/2$, $D=10$ $(n_{1/2^+}=32)$; 3) $N=1_{+-}$, $D=10$; 4) $N=1_{++}$, $D=2(n_{1/2^+}=48)$; 5)$N=3/2_{++-}$, $D=2(n_{1/2^+}=24)$; and 6) $N=2_{++--}$,$D=2$. Chiral theories 2), 4), and 5) may be interpreted as "chiral combinations" of theories[2] 1), 3), and 6) (if 48 chiral scalars are substituted for 24 chiral spinors, for example, we can consider $N=3/2_{++-}$ theory as a combination of the "left" part of the $N=2_{++--}$ theory and the "right" part of the $N=1_{+-}$ theory[2].

The condition for the cancellation of the $d = 2$ anomalies in the *free* action of the string imposes only a crude constraint on the structure of the theory. More rigorous constraints may arise in the analysis of the cancellation of the $d = 2$ anomalies in the action of the string which interacts with the external fields (in terms of the effective action suggested in Ref. 12). If the $N = 1, D = 10$ supergravity and Yang-Mills boson fields are connected to the action of the heterotic string, for example, we find (in a tree approximation in the string theory, i.e., in S^2) that the corresponding $d = 2$ chiral and Lorentz anomalies can be cancelled by assuming that the antisymmetric tensor $B_{\mu\nu}$ transforms under $D = 10$ Lorentz and gauge transformations according to the law which was found in Ref. 1 from the condition for the cancellation of the $D = 10$ anomalies (see also Ref. 13). We can assume that the condition for the cancellation of the $d = 2$ anomalies in a torus (which corresponds to a one-loop approximation in the string theory) accounts for the constraints imposed on the gauge group of the "external" gauge field [$E_8 \times E_8$ or SO(32)] and also possibly on the structure of the ground state of the theory.

[1)] A highly simplified supersymmetry algebra in $d = 2$ has a Majorana-Weyl generator (see Refs. 3 and 4, for example). N denotes the total number of such generators; $N = 2_{+++-}$, for example, corresponds to a theory with three "left" supergenerators and a single "right" supergenerator.

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