# Theory of edge magnetoplasmons in a two-dimensional electron gas

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In a bounded two-dimensional electron gas the spectrum of edge magnetoplasmons is a gap-free spectrum. In the quantum Hall effect regime, the velocity of the "optical" edge magnetoplasmons is quantized. In classical magnetic fields, the results are in quantitative agreement with the experimental data for 2D electrons bound to a liquid-helium surface.

Plasma oscillations in an unbounded 2D electron gas, in contrast with a 3D electron gas, obey a gap-free dispersion law[1]:

$$\omega_p(q_x, q_y) = \left[\frac{4\pi n_s e^2}{m} \, \frac{|\mathbf{q}|}{\epsilon_s + \epsilon_i \coth |\mathbf{q}| d}\right]^{1/2}, \qquad (1)$$

where $\mathbf{q} = (q_x, q_y)$ is the wave vector of a 2D plasmon, $n_s$ and $m$ is the density and the effective mass of electrons, $d$ is the thickness of the insulator in an MIS structure, and $\epsilon_s$ and $\epsilon_i$ are the dielectric constants of the semiconductor and insulator. In a magnetic field $\mathbf{H}$ perpendicular to the 2D layer, the magnetoplasmon spectrum acquires a gap equal to the cyclotron frequency $\omega_c = eH/mc$ at $\mathbf{q} = 0$:

$$\omega_{mp}(q_x, q_y) = [\omega_c^2 + \omega_p^2(q_x, q_y)]^{1/2}. \qquad (2)$$

Studies of the quantum Hall effect have established the importance of edge electronic states near the lateral boundaries of the inversion layer.[2] In this letter we will show that these edge states lead to the appearance of gap-free edge magnetoplasmons which have recently been detected experimentally by Glattli *et al.*[3] and Mast *et al.*[4] We will show that an exact solution of the edge magnetoplasmon spectrum given below differs markedly from that obtained in Refs. 4 and 5 by means of uncontrollable approximations.

Let us consider a semi-infinite inversion layer occupying a region $x \geqslant 0$, $z = 0$, $-\infty < y < +\infty$ in a magnetic field $\mathbf{H} = (0, 0, H)$. The insulator and semiconductor occupy the regions $0 < z < d$ and $z < 0$, respectively. We seek the edge magnetoplasmon potential in the form

$$\varphi(x, y, z, t)\,|_{z=0} = \varphi(x) \exp(-i\omega t + iq_y y). \tag{3}$$

Ignoring the retardation, we find in the random-phase approximation a system of integral equations for the determination of $\varphi_{NN'}(X)$ matrix elements of $\varphi(x)$ from the wave functions $|N, X\rangle$, where $N$ is the number of the Landau level $E_N(X)$, and $X$ is the $x$-center of the oscillator. In the limit of strong $H$, in which the matrix elements (which are nondiagonal in $N$) of the electron-electron-interaction potential $V_{N'N}$ can be ignored, under the conditions $\omega \ll \omega_c$, $|q_y|\lambda \ll 1$, and $T = 0\,K$ we can greatly simplify the system of equations:

$$\varphi_{MM}(X) = \frac{e^2 q_y}{\pi\hbar} \sum_N \frac{\varphi_{NN}(X_{FN})\, V_{MN}(X)}{\omega - q_y v_{FN} + i0}. \tag{4}$$

Here $\lambda$ is the magnetic length, $\hbar v_{FN} = -\lambda^2 \partial E_N(X_{FN})/\partial X$, $X_{FN}$ is the root of the equation $E_N(X) = E_F$, and $E_F$ is the Fermi level. Setting $X = X_{FM} \approx 0$ in (4), we find for $\omega \gg |q_y v_{FN}|$ and for completely filled $N$ Landau levels the spectrum of the "optical" edge magnetoplasmons

$$\omega(q_y) = \frac{e^2 N q_y}{\pi\hbar} L(0, q_y), \tag{5}$$

$$L(x, q_y) = \frac{2}{\epsilon_s + \epsilon_i}\left\{ K_0(|q_y|\sqrt{x^2 + l^2}) - \frac{2\epsilon_i}{\epsilon_s + \epsilon_i} \sum_{n=1}^{\infty} \left(\frac{\epsilon_s - \epsilon_i}{\epsilon_s + \epsilon_i}\right)^{n-1} K_0(|q_y|\sqrt{x^2 + (2dn + l)^2}) \right\}, \tag{6}$$

where $K_0(x)$ is the MacDonald function, and $l$ is the cutoff length that will be calculated below. Let us consider the asymptotic behavior of (5) for $|q_y|l \ll 1$

$$\omega(q_y) = \frac{2e^2 N q_y}{\pi\hbar(\epsilon_s + \epsilon_i)} \left[\ln \frac{1}{|q_y| l} + O(1)\right], \quad |q_y| d \gg 1; \tag{7}$$

$$\omega(q_y) = \frac{e^2 N}{\pi\hbar\epsilon} q_y \ln((2d + l)/l), \quad |q_y| d \ll 1, \quad \epsilon_s = \epsilon_i = \epsilon. \tag{8}$$

For $N > 1$, aside from the "optical" branch of the edge magnetoplasmons in (5), there are also $N-1$ "acoustic" branches with phase velocities $S_i$, where $v_{F,i} < S_i < v_{F,i+1}$.

The nondiagonal elements of $V_{N'N}$ form the cutoff length $l$. This can be shown by analyzing the phenomenological model of the inversion layer which is characterized by the local conductivity tensor $\sigma_{\alpha\beta}(\mathbf{r},\omega) = \sigma_{\alpha\beta}(\omega)\delta(z)\theta(x)$, where $\alpha,\beta = x,y$, $\theta(x) = 1$ for $x > 0$, and $\theta(x) = 0$ for $x < 0$. From a self-consistent system of equations (equations of continuity, Poisson equations, constitutive equations) for potential (3) we can find, in the same way as in Refs. 4 and 5, the integral equation

$$\varphi(x) + \int_0^\infty k(x-x_0)\varphi(x_0)\,dx_0 = f(x), \tag{9}$$

where

$$k(x) = \frac{2i\sigma_{xx}}{\omega}\left(-\frac{\partial^2}{\partial x^2} + q_y^2\right)L(x,q_y);\; f(x) = -\frac{2\varphi(0)}{\omega}\left[q_y\sigma_{xy} + i\sigma_{xx}\frac{\partial}{\partial x}\right]L(x,q_y).$$

An equation that reduces to (9) was solved in Refs. 4 and 5 by means of an uncontrollable approximation (by substituting an exponential kernel for a complex kernel). Equation (9) can be solved exactly by the Wiener-Hopf method.[6] The Fourier transform of the function $\theta(x)\varphi(x)$

$$\phi_+(q_x) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{dq_x'}{q_x' - q_x - i0}\,\frac{G_+(q_x)}{G_-(q_x')}\int_0^\infty dx\,e^{iq_x'x} f(x) \tag{10}$$

can be expressed in terms of the function $G_\pm(q_x)$ and in terms of the dielectric constant $\epsilon(\mathbf{q},\omega)$:

$$G_\pm(q_x) = \exp\left\{-\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{dQ}{Q - q_x \mp i0}\ln\epsilon(Q,q_y,\omega)\right\},$$
$$\epsilon(q_x,q_y,\omega) = 1 + \frac{4\pi i\sigma_{xx}}{\omega}\,\frac{|\mathbf{q}|}{\epsilon_s + \epsilon_i\coth|\mathbf{q}|d}. \tag{11}$$

Since the function $\phi_+(q_x)$ has no singularities in the upper half-plane, its inverse Fourier transform is $\phi_+(x) = \varphi(x)$ for $x > 0$ and $\phi_+(x) = 0$ for $x < 0$. The value $\phi_+(x=0) = \varphi(0)/2$ can be simplified substantially. After a reduction to $\varphi(0)$, we find an equation for the spectrum of "optical" edge magnetoplasmons

$$1 + \frac{q_y\sigma_{xy}}{i|q_y|\sigma_{xx}}\tanh\left\{\frac{1}{\pi}\int_0^\infty\frac{dQ}{1+Q^2}\ln\epsilon(|q_y|Q,q_y,\omega)\right\} = 0. \tag{12}$$

The edge magnetoplasmon spectrum is comprised of two branches, $\omega_+$ and $\omega_-$, indexed by $q_y$ (Fig. 1). Let us consider the asymptotic behavior of $\omega(q_y)$ found after substitution of (11) into (12). In the case $|q_y|d \gg 1$, the right gap-free branch and the left branch with a gap can be written in the form

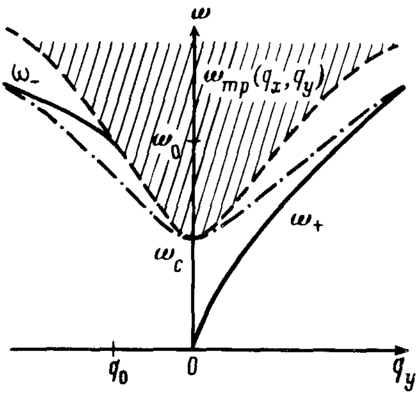


FIG. 1. Spectrum of the edge magnetoplasmons (solid line). Dot-dashed curve is the curve $\omega^2 = \omega_c^2 + [0.906\times\omega_p(0, q_y)]^2$ which describes the edge magnetoplasmon spectrum at $H = 0$; hatching denotes the continuous spectrum.

$$\omega_+(q_y) = -\frac{4q_y\sigma_{xy}}{\epsilon_s + \epsilon_i}\ln\frac{2{,}72}{|q_y|l}, \quad \omega \ll \omega_c; \tag{13}$$

$$\omega_\pm(q_y) = 0.906\omega_p(0, q_y)\left\{1 - 0.390\frac{\omega_c\,\mathrm{sign}\,q_y}{\omega_p(0, q_y)} + \ldots\right\}, \quad \omega \gg \omega_c; \tag{14}$$

$$l = \frac{2\pi}{\epsilon_s + \epsilon_i}\left.\frac{i\sigma_{xx}(\omega)}{\omega}\right|_{\omega\to 0} = \frac{2\pi n_s\lambda^2}{\epsilon_s + \epsilon_i}\frac{e^2}{\hbar\omega_c}. \tag{15}$$

The second expression for the cutoff length $l$ is written in the Drude model. The branch $\omega_-(q_y)$ is included in the continuous spectrum $\omega_{mp}(q_x, q_y)$ for $q_y = q_0 = -1.315/l$ and $\omega_0 = 1.905\omega_c$. In the cases $|q_y|d \ll 1$ and $\omega \ll \omega_c$ $(\epsilon_s = \epsilon_d = \epsilon)$, we have

$$\omega_+(q_y) = -\frac{2q_y\sigma_{xy}}{\epsilon}\ln(4{,}841\,d/l), \quad d \gg l; \tag{16}$$

$$\omega_+(q_y) = -\frac{2\pi\sigma_{xy}q_y}{\epsilon}\sqrt{d/l}, \quad d \ll l. \tag{17}$$

In the quantum Hall effect regime $\sigma_{xy}(0)$ is a multiple of $e^2/2\pi\hbar$ and (13), (16), and (17) imply quantization of the phase velocity of the right branch of the edge magnetoplasmons; (13) and (16) are consistent with (7) and (8). Expression (17) was obtained by Glattli *et al.*[3] in the weak $H$ approximation and is consistent with their experimental data. The experimental data of Ref. 4 are in the region where (16) applies; here $\sigma_{xy}$ is described by the Drude model. Figure 2 is a plot of the frequency of the edge magnetoplasmon as a function of the mode number $n = q_yP/2\pi$ (conversion of the experimental data of Ref. 4 for 2D electrons bound to a liquid-helium surface for $n_s = 2\times10^8$ and $2.8\times10^8$ cm$^{-2}$; $P = 8.56$ cm is the perimeter of the sample). Only the even modes are seen at small values of $n$, which Mast *et al.*[4] attribute to the particular features of the excitation of the edge magnetoplasmons (the arrow indicates

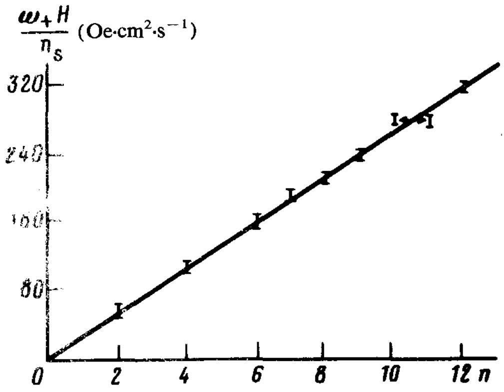


FIG. 2

the ambiguity in the determination of certain values of $n$ for $n \gg 1$). The dependence $\omega_+(q_y)$ is clearly a linear dependence, consistent with (16) but inconsistent with the theory of Mast *et al.*[4] and Wu *et al.*[5] From a comparison with (16), where $\sigma_{xy} = -en_s c/H$ and $d$ for a system with two equidistant shutters[4] is equal to one-half the distance to the shutter, we can determine the cutoff length: $l \approx 1.4d$. This distance is not determined by expression (15) which is obtained within the limit of an abrupt change in $\sigma_{\alpha\beta}$ at $x = 0$ but instead by a much greater transition-layer thickness[3] $n_s(x)$.

At $\omega < \omega_0$ the presence of edge magnetoplasmons with just one velocity direction leads to a considerable suppression of the standard collisional mechanism of plasmon damping, which accounts for the sharp narrowing (by an order of magnitude, according to Mast *et al.*[4]) of the absorption line of the edge magnetoplasmons in comparison with the "bulk" modes.


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