Effective chiral Lagrangians in QCD and the skyrmion

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The self-effect of pseudoscalar mesons does not lead to the existence of locally stable solitons as nucleons. The mass of the chiral soliton which is stabilized when the interaction with vector ω and ρ mesons is taken into account is roughly twice the mass of a nucleon.

Interaction models of the Skyrme type[1] predict solitons with the spin and mass of a baryon when the multivalued Wess-Zumino action is taken into account. This action must be taken into account in order to correctly reproduce the properties of quantum chromodynamics (QCD) upon CP reflection. In the present letter we analyze soliton-like solutions with the properties of baryons for a realistic effective Lagrangian,[2] which describes the fluctuations of the γ_5 phase in QCD, found by the method of chiral bosonization. We also analyze several of its phenomenological modifications with ω and ρ mesons.

The effective low-energy Lagrangian[2] for the chiral phase of quarks contains all terms of fourth order in the fields and their derivatives. This Lagrangian is thereby distinguished from the Lagrangian of the standard chiral model. The coefficient of the invariant structures are determined by the number of colors, $N_c = 3$, and by the low-energy scale of QCD, Λ. If we restrict the discussion to massless up and down quarks, i.e., a chiral field $U \in SU(2)$, we can conveniently write the effective Lagrangian in the form $L^{\text{eff}}(U) = L^{\text{Sk}}(U) + \delta L(U)$, where

$$L^{\text{Sk}}(U) = \frac{N_c \Lambda^2}{16\pi^2} \operatorname{tr} \partial_\mu U \partial^\mu U^+ + \frac{1}{128\pi^2} \operatorname{tr} [\partial_\mu U U^+, \partial_\nu U U^+]^2 \tag{1}$$

describes the Lagrangian of the Skyrme model[1] with $e = 2\pi$ and $N_c \Lambda^2/16\pi^2 = F_\pi^2/4, F_\pi$- is the constant of the pion decay, equal to 93 MeV. The expression

$$\delta L(U) = \frac{1}{64\pi^2} \operatorname{tr} [(\partial_\mu U \partial^\mu U^+)^2 - 2(\partial_\mu (\partial^\mu U U^+))^2] \tag{2}$$

contains terms which disrupt the positive definiteness of the energy functional. The last term in (2), which contains second derivatives, strongly changes the π-meson propagator, $1/p^2 \to 1/p^2 + p^4/\mu^2$, where the quantity $\mu^2 = 24\pi^2 F_\pi^2/N_c \cong$ (830 MeV)^2 is approximately equal to the mass of the ρ meson. We thus find a ghost particle: a tachyon, which restricts the range of applicability of the effective Lagrangian $L^{\text{eff}}(U)$. In the fields U for which the inequality

$$\langle p^4 \rangle / \langle p^2 \rangle \ll \mu^2 \tag{3}$$

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holds, the additional terms in (2) can be regarded as small perturbations of the Skyrme model.

For a description of low-lying states of nucleons, we are interested[1] in spherically symmetric soliton configurations with baryon number $B = 1$, of the "porcupine" type: $U(x) = \exp\{iF(r)\cdot\vec{\tau}\hat{\mathbf{x}}\}$, where $F(0) = \pi$ and $F(r) \to 0$ in the limit $r \to \infty$. For the effective Lagrangian $L^{\text{eff}}(U)$ the mass functional for the variable $\tilde{r} = 2\pi F_\pi r$ is

$$M^{\text{eff}}[F] = M^{\text{Sk}}[F] + 2F_\pi \int_0^\infty d\tilde{r} \left[\frac{\sin^4 F}{2\tilde{r}^2} + \frac{3}{2} \sin^2 F F'^2 - \frac{1}{4}\tilde{r}^2 F''^2 - \frac{1}{8}\tilde{r}^2 F'^4 \right], \quad (4)$$

where $M^{\text{Sk}}[f]$ is the known[1] mass functional of a static soliton in the Skyrme model; this functional is positive definite and has a lower limit. It can be seen from (4) that $M^{\text{eff}}[F]$ loses both its positive definiteness and its lower limit. Nevertheless, there is the hope that locally stable solitons may exist within the range of applicability of the effective Lagrangian. A variational study of (4) with the help of Padé approximants shows that the value of the mass functional has no lower limit, that there is no local minimum on static configurations, and that inequality (3), which defines the range of applicability of the Lagrangian $L^{\text{eff}}(U)$, does not hold on soliton configuration with $B = 1$. The disruption of the positive definiteness of the functional $M^{\text{eff}}[F]$ is due in large extent to the negative contribution of the tachyon term. At low energies in the massless limit, using classical equations of motion, we find $\partial_\mu^2 U \partial_\nu^2 U^+ = (\partial_\mu U \partial^\mu U^+)^2$, which leads to a cancellation of the tachyon term, so that the second term in (2) vanishes in the semiclassical approximation. In this case the mass functional is

$$\tilde{M}[F] = M^{\text{Sk}}[F] - \gamma F_\pi \int_0^\infty d\tilde{r}\,\tilde{r}^2 \left[F'^2 + \frac{2\sin^2 F}{\tilde{r}^2} \right]^2, \quad \gamma = \frac{1}{4}. \quad (5)$$

A variational study of (5) shows that it is greater in magnitude than in the model in (4), and it has become positive definite over a broad region of the solution of the Skyrme model. On the whole, however, the mass functional does not become positive definite, and there is no local minimum in the region over which the parameters can be varied. Furthermore, there is no justification for the existing approach[3] of treating the last term in (5) as a small perturbation in the Skyrme model. Studies of Skyrme solutions show that this term is not small, so that inequality (3) does not hold. Variational studies of the functional $\tilde{M}[F]$ outside perturbation theory, for an arbitrary positive $\gamma > 0$, indicate that there is no locally stable soliton at $\gamma \gtrsim 0.12$. Consequently, the realistic meson Lagrangian L^{eff} and its modification do not lead to stable solitons with the quantum numbers of baryons.

Let us supplement the chiral models discussed above with an interaction with a vector ω meson by the decay $\omega \to 3\pi$ (Ref. 4), in an effort to stabilize the soliton. In the approximation of heavy ω mesons we find the following mass functional for the contribution of the ω mesons:

$$\Delta M_\omega[F] = \frac{4\beta^2 F_\pi^3}{m_\omega^2} \int_0^\infty d\tilde{r}\, \frac{\sin^4 F}{\tilde{r}^2} F'^2, \quad (6)$$

TABLE I.

β	M^{Sk} MeV	$M^{Sk}+\Delta M_\omega$ MeV	$M^{eff}+\Delta M_\omega$ MeV	$\tilde{M}+\Delta M_\omega$ MeV
15	1080	1803	–	1591
20	1080	2017	1622	1827
25	1080	2207	1858	2036

where m_ω is the mass of the ω meson, and β is the constant of the decay $\omega \to 3\pi$. A variational study of mass functionals with an ω meson in M^{Sk},M^{eff}, and $\tilde{M}$ has revealed local minima of these mass functionals. The results are given in Table I for various values of the decay constant β.

Let us evaluate the contribution of the ρ meson to the mass functionals by incorporating that vertex of the decay $\rho \to \pi\pi$ which is the smallest in terms of the number of derivatives. In the approximation of heavy ρ mesons we find the following result for the mass functional:

$$\Delta M_\rho = -\frac{F_\pi^4 g_{\rho\pi\pi}^2}{32 m_\rho^2} \int \mathrm{Tr}\,[U^+, \partial_\mu U]^2\, d^3x. \tag{7}$$

Although the ρ meson lowers the mass of the soliton, the change ΔM_ρ is small, $\lesssim 1\%$ in comparison with ΔM_ω on the static configurations considered. Consequently, incorporating the interaction with vector ω and ρ mesons stabilizes the baryon soliton, but its mass is significantly greater than (roughly twice) the mass of a nucleon.

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