Final-state interaction of a β particle and associated phenomena

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The final-state interaction of a β particle with atomic electrons and corrections to the lifetime of atomic tritium and pionium are calculated to terms of order α^2/v^2. The final-state interaction makes it possible to extract new information on the electron distribution near the T in a medium. When the final-state interaction is taken into account, Simpson's data cannot be explained by the emission of a heavy neutrino.

Sixteen high-precision measurements of the β spectrum in the decay of tritium have now been carried out in an effort to determine the rest mass of the electron neutrino within ~ 1 eV (see the review by Vanucci[1]). This process has also been used recently in a search for the mixing of ν_e with heavy neutrinos.[2] Since the energy released in the decay is small, $E_0 = 18.6$ keV, an analysis of the data—especially for experiments of the type in Ref. 2—requires a careful account of the interaction of the β particle with the residual atom in the final state. While the interaction of the β particle with the daughter nucleus can be described by a standard Fermi function, calculations on the final-state interaction with a bound electron require a series expansion in the Coulomb parameter $\zeta = \alpha/v$, where v is the velocity of the β particle. In the soft part of the β spectrum ($E_e \lesssim 1$ keV), this correction is substantially larger than the error ($\sim 10^{-3}$) in the measurements of the β spectrum.[2] This correction changes significantly if tritium is implanted in a medium, so that measurements of the β spectrum in the soft region offer a way to study the tritium wave function in a medium (another way is to study the change in the lifetime of a π^+ meson in a medium). The information which arises here complements that obtained from μSR analyses.

The leading corrections ($\sim \zeta^2$) in the final-state interaction were studied in Ref. 3, while corrections $\sim \zeta^4$ from the interchange of the atomic electron with the β particle were recently studied in Ref. 4. In the present letter we show that even a calculation of corrections[3] $\sim \zeta^2$ is unsatisfactory, since it does not take into account all the diagrams that make contributions $\sim \zeta^2$. Only the first diagram of a perturbation theory in the final-state interaction was taken into account in Ref. 3. The leading contribution to the amplitude turns out to be imaginary:

$$\mathrm{Im} F^{(1)} \sim \zeta; \quad \mathrm{Re} F^{(1)} \sim \zeta^2. \tag{1}$$

Contributions are made to the probability by both the product $F^{(0)}\mathrm{Re}F^{(1)} \sim \zeta^2$ ($F^{(0)}$ is the real "shaking" amplitude[6]) and $|\mathrm{Im}F^{(1)}|^2 \sim \zeta^2$. On the other hand, the twofold interaction of the β particle with the bond electron (Fig. 1b), $F^{(2)}$, also contains a real part $\sim \zeta^2$. Furthermore, a summation of the square amplitude over the final states of the atom results in the relation

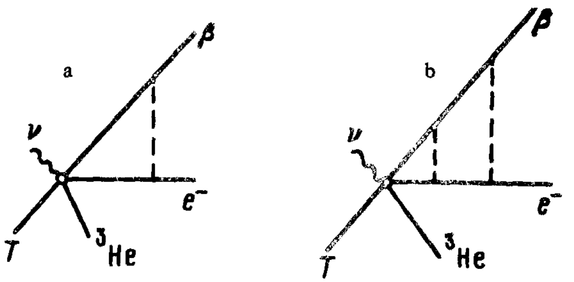

FIG. 1.

$$2F^{(0)}\mathrm{Re}F^{(2)} + |\mathrm{Im}F^{(1)}|^2 = 0, \tag{2}$$

which effectively cancels out the infrared divergences. Consequently, the square amplitude incorporating the final-state interaction is

$$|F|^2 = |F^{(0)}|^2 + 2\,\mathrm{Re}F^{(1)}F^{(0)}. \tag{3}$$

Calculating $\mathrm{Re}F^{(1)}$ in the standard way, using the diagrams in Fig. 1a, we find the following expression for the change in the energy distribution of β particles due to the final-state interaction:

$$\frac{dW/dE}{dW^{(0)}/dE} = 1 - \zeta^2 < \chi | \frac{r_0}{r} | \chi > , \tag{4}$$

where $r_0 = 1/m\alpha$ is the Bohr radius, and $|\chi\rangle$ is the wave function of the initial state of the atom (for a free tritium atom, $\langle\chi|\frac{r_0}{r}|\chi\rangle = 1$). Expression (4) is easily generalized to the case of an atom with z electrons[1] [under the condition $(\alpha z/v)^2 \ll 1$)],

$$\frac{dW/dE}{dW^{(0)}/dE} = 1 - \zeta^2 \langle\chi| \sum_i^z \frac{r_0}{|r_i|} |\chi\rangle, \tag{5}$$

and to the case in which the state of the residual atom is fixed,

$$\frac{dW_n/dE}{dW_n^{(0)}/dE} = 1 + \zeta^2 a_n / K\chi|n\rangle|^2, \tag{6}$$

$$a_n = -\langle\chi|n\rangle\langle n|\frac{r_0}{r}|\chi\rangle + \sum_k \frac{-E_n + E_k}{m\alpha^2}\langle\chi|n\rangle\langle n|\frac{r}{r_0}|K\rangle\langle K|\chi\rangle$$

$$+ \frac{1}{4}\left[|\langle n|\ln\frac{r_t^2}{\lambda^2}|\chi\rangle|^2 - \langle\chi|n\rangle\langle n|\ln^2\frac{r_t^2}{\lambda^2}|\chi\rangle\right].$$

The numerical values here are $a_1 = -1.55$, $a_2 = 0.49$, and $a_3 = 0.035$ (λ is a cutoff parameter, on which a_n does not depend). In Ref. 4, the coefficients a_n were overestimated by a factor of about three, so that the corrections for the final-state interaction in the measurements were small at $m_{\nu_e} \gtrsim 0.1$ eV.

In an analysis of the experiment of Ref. 2, it is necessary to take into account the circumstances that (a) the semiclassical procedure for taking the final-state interaction in Ref. 2 into account is qualitatively at odds with Eq. (4) and (b) there is a change in the value of $1/r$ in a medium.[2)] It follows from data on a μSR analysis for Si that in 60% of the cases muonium either does not form in the medium or is in an anomalous state, in which $|\psi(0)|^2$ is very small. In other cases, $|\psi(0)|^2$ is also substantially suppressed by a factor ~ 0.4 (see the review by Gordeev and Obukhov[7]). As a result, we can expect

$$\langle \chi | r_0/r | \chi \rangle \sim 0.3. \tag{7}$$

An accurate evaluation of this quantity is a problem in its own right.

We might note two other applications of Eq. (4): (a) a correction to the tritium lifetime (-0.51%), which substantially changes the results of the analysis of Ref. 8; (b) a change in the lifetime of a π^+ meson in a medium due to the formation of pionium,

$$\tau^{\pi^+}_{\text{bound}} / \tau^{\pi^+}_{\text{free}} = 1 - \frac{m_e}{m_\mu} \frac{\alpha^2}{v_\mu^2} \langle \chi | \frac{r_0}{r} | \chi \rangle \cong 1 - 4 \times 10^{-6}. \tag{8}$$

We see that this correction is much smaller than the accuracy ($\sim 2\times 10^{-4}$) which has been achieved in the measurements of τ^{π^+} on the basis of the decays of π^+. These decays are produced in proton-nucleus interactions and are stopped in the same target.[9] Consequently, the effects of the medium do not yet limit the accuracy of the τ^{π^+} measurements.

Using the decays of T implanted in a medium, using Eq. (1), and measuring the spectrum of β particles at $E_e \sim 1$ keV, we can in principle measure $\langle \chi | \frac{r_0}{r} | \chi \rangle$ in a medium. Obviously, this method for extracting information on the electron distribution near a charge implanted in a medium complements the μSR method.

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[1)]The incorporation of the final-state interaction during ionization in the case of the β decay of heavy nuclei has been studied in several places (see the bibliography in the review by Batkin and Smirnov[5]), but the contribution of $F^{(2)}$ (Fig. 1b) has been disregarded. The corollaries of Eq. (5) will be discussed in a separate paper.

[2)]We recall that the interpretation of the data of Ref. 2 as a mixing of ν_e with a heavy neutrino has been refuted by experiments on the β decay of ^{35}S (Ref. 6), where the effects of the final-state interaction were negligibly small. That interpretation is also incompatible with our model of the final-state interaction.

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