Antisymmetric tandem mirror

A. A. Skovoroda

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It may be possible to devise a locally nested classical tandem mirror confinement system in which the axial profile of the quadrupole component of the magnetic potential is antisymmetric with respect to the center.

The use of ordinary *minB* quadrupole confinement systems as the end mirror cells of a tandem mirror (or "ambipolar confinement system") has led to negative consequences.[1] The significant deviation from nesting and the splitting of the drift shells increase the transverse transport and give rise to longitudinal currents in the uniform part of the tandem mirror. As a way to reduce these effects and to automatically satisfy the condition of "global" nesting,[1,2] there has been a discussion of tandem mirrors in which the multipole component (g) of the magnetic potential has a z profile which is symmetric with respect to the center ($z=0$): $g(z)=g(-z)$. Devices of this type are in fact being constructed. In the paraxial approximation, the magnetic field of a quadrupole confinement system can be described in cylindrical coordinates r, ϕ, z by the magnetic potential[1]

$$\Phi = \int^{z} B_0 dz + \left(-\frac{1}{4}\frac{dB_0}{dz} + g\cos 2\phi \right) r^2 , \tag{1}$$

where B_0, the magnetic field at the axis, and g, the quadrupole component, are independent functions of z. A condition of local orthogonality (nesting) has been derived in several studies[1,3]:

$$[\,\mathbf{b}\nabla B]\,\nabla(\mathbf{b}\nabla B\,) = 0, \tag{2}$$

where B is the modulus of the magnetic field ($\mathbf{B}=\mathbf{b}B$). Condition (2) has been analyzed[1–3] in the paraxial approximation for *symmetric g* profiles.

Let us examine the basic results of this analysis. For an isolated confinement system we can use the given profile $B_0(z)$ (which has one minimum) to find the required functional dependence $g(z)$ for stability and local orthogonality of the confinement system. Despite the engineering complexities in actually constructing such magnet systems, the problem is solvable in principle. For a tandem mirror with a $B_0(z)$ profile having several minima, it is possible to find a solution only with $g\neq 0$ along the entire central cell, which would make the end mirror cells MHD-unstable. In a system of this sort, in contrast with the classical tandem mirror, the stability of the overall system is maintained by the central cell, rather than by the end cells. This circumstance seriously restricts the length of the central cell. The analysis that has been carried out thus does not give us a solution, in principle, for the problem of devising a locally nested *classical* tandem mirror.

In the present letter we discuss the possibility of developing a locally nested classical tandem mirror with an *antisymmetric* profile: $g(z) = -g(-z)$.

Let us write condition (2) in the approximation of a slight multipolarity, in which we can write the magnetic potential as $\Phi(\mathbf{r},\phi,z) = \Phi^0(r,z) + \epsilon\Phi^1(r,\phi,z)$, where $\epsilon \ll 1$. We find

$$\Phi^1_{z\phi}\Phi^0_{zzr} - \Phi^1_{zz\phi}\Phi^0_{zr} = 0, \tag{3}$$

where the subscripts mean differentiation. There is a wide class of functions Φ that satisfy (3) for all r, ϕ. In particular, using (1) in the paraxial approximation we can find

$$g(z) = \pm\alpha\frac{dB_0}{dz}, \tag{4}$$

where α is an arbitrary positive constant. It is easy to show that this antisymmetric solution forms MHD-unstable configurations. The drift shells are closed if $\alpha < 1/4$, and in their central cross section they are elliptical with an axis ratio $\sqrt{(\alpha + 1/4)/(\alpha - 1/4)}$. Numerical calculations on the drift motion of particles in model magnetic fields with small mirror ratios ($\leqslant 3$) reveal a good nesting under condition (4).

Let us use solution (4) in the central cell of a tandem mirror. For this purpose, we use the paraxial approximation to calculate the profile $g(z)$ [Eq. (49) in Ref. 3] for the orthogonality in the end cell of a tandem mirror (regions 1 and 2), and we then join these profiles at the center (regions 3 and 4) by means of solution (4) (Fig. 1). Figure 1 shows one such design. The field in the end cell (region 1) is modeled by a very simple function (see the Fig. 1 caption). Transition region 2 is approximated by a polynomial whose constants are chosen in such a way that the calculated solution $g(z)$ satisfies the boundary conditions$^{1)}$ $g(z_2) = \alpha B_0'(z_2), g'(z_2) = \alpha B_0''(z_2)$. The field in transition region 3 is also approximated by a polynomial, chosen in such a way that the condition

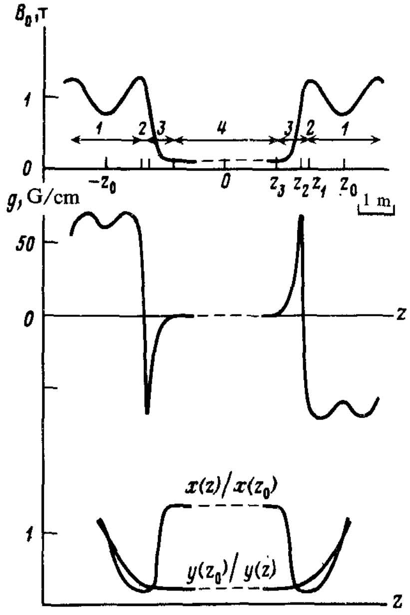

FIG. 1. "Antisymmetric" tandem mirror. $x(z)$, $y(z)$—Positions of lines of force in the xz and yz planes. The field in region 1 is $B_0(z)/B_0(0) = 1 - \nu\cos(\pi z/L)$, where $\nu = 0.25$ and $L = 100$ cm.

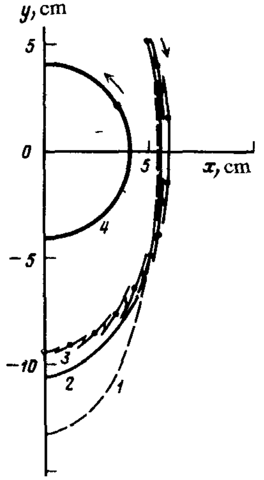

FIG. 2. Projections of the drift trajectory of a hydrogen ion in a tandem mirror along magnetic lines of force. 1—Intersection of a magnetic surface with the $z = 0$ plane; 2—projection onto the $z = 0$ plane for an ion with an energy of 1 keV (the longitudinal velocity is $v_{\parallel} = 10^8$ cm/s; 3—again with $v_{\parallel} = 2 \times 10^8$ cm/s; 4—projection and intersection of the magnetic surface with the $z = z_0$ plane for ions with an energy of 10 keV and $v_{\parallel} = 4 \times 10^7$ cm/s. The arrows indicate the drift direction; the points on the curves are the intersection of particles with planes.

$B_0''(z_3) = 0$ holds. It thus becomes possible to insert a uniform field region of arbitrary length (region 4) in the center of the tandem mirror. The value of α and the mirror ratio in the central cell are chosen to bring the orthogonal magnetic surfaces[3] in the central and end cells into coincidence.

Figure 2 shows the calculated projections of the drift trajectories of the particles in the central and outermost cross sections of the tandem mirror. Curve 2 corresponds to an ion that is confined in the central cell, which has a mirror ratio of 9.62 (high). We see that at high mirror ratios (~ 10) a deviation from nesting arises, but this deviation is smaller than or comparable to the ion Larmor radius and thus could not cause any significant transport.

There is essentially no splitting of a drift shell. Curve 4, which corresponds to an ion confined in an end cell, demonstrates the good nesting. Curve 3 corresponds to an ion confined by the potential in an end cell. This ion is bouncing essentially between the points $(-z_0, z_0)$. In the calculations the potential is introduced in model form, as a mirror wall at the point of the minimum on a line of force. We see the appearance of a splitting of a drift shell; this splitting, which is considerably smaller than the ion Larmor radius, is determined by the noncoincidence of the magnetic surfaces at the center and in the end cells (this discrepancy is 10% in this particular calculation). A more careful calculation might eliminate this discrepancy, but the point may be irrelevant, since the number of particles that undergo this small splitting is exponentially small, so that only small longitudinal currents will arise. Calculations show that the picture outlined here changes only slightly if a small region, which is nonorthogonal but which allows us to make the profiles $B_0(z)$ and $g(z)$ analytic, is inserted between regions 2 and 3.

In summary, the neoclassical losses, the longitudinal currents, and the multipole component in the central cell of a tandem mirror can be eliminated in this new design. It would thus become possible, in particular, to eliminate the restrictions on the length of the central cell.

$^{1)}$The prime means the derivative with respect to z.

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