Aaronov-Bohm oscillations with normal and superconducting flux quanta in hopping conductivity

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It is shown that in the hopping-conductivity region there are oscillations in the resistance as a function of the magnetic flux spanning the sample (the Aaronov-Bohm effect). The oscillation period may be equal to the normal or superconducting flux quantum. The transition between the two regimes is a second-order phase transition.

Al'tshuler *et al.*[1] showed that the resistance of a thin-walled cylinder of a dirty metal should oscillate as a function of the magnetic flux penetrating the cylinder with a period of $\cosh/2e$, half the normal flux quantum $\Phi_0 = \cosh/e$. Oscillations of this type were discovered by Sharvin and Sharvin.[2] According to Ref. 1, at sufficiently low temperatures the oscillation amplitude should increase as the metal becomes dirtier. We are thus led to ask whether resistance oscillations with a period of $\Phi_0/2$ exist on the insulator side of the metal-insulator transition, i.e., in the region of hopping conductivity. In this letter we show that in this region there can be oscillations with periods of both Φ_0 and $\Phi_0/2$, and the two regimes will be separated by a second-order phase transition in the impurity concentration or degree of compensation. This transition is a reflection of a phase transition which we have observed in the sign structure of the Green's function. Although the latter transition is general in nature, we will demonstrate it for the Anderson model on a square lattice with $(n+1)^2 = N$ sites (Fig. 1), with the Hamiltonian

$$H = \sum_{i=1}^{N} \epsilon_i a_i^+ a_i + \sum_{i \neq j} V_{ij} a_i^+ a_j, \tag{1}$$

where V_{ij} is equal to V for nearest neighbors and 0 otherwise; the energies of the extreme left and extreme right sites, ϵ_1 and ϵ_N, are approximately zero; and the other energies ϵ_i take on the values $-W$ and W in a random manner with probabilities of x and $1-x$, respectively. Here $W>0$ and $W \gg |V|$. We seek the effective overlap integral I between sites 1 and N:

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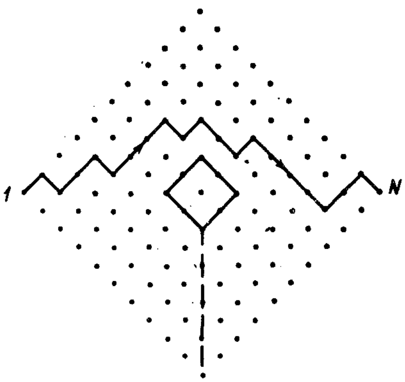

FIG. 1.

$$I = V \sum_{\{\Gamma\}} \prod_{\{i_\Gamma\}} \left(\frac{V}{\epsilon - \epsilon_i} \right) \Bigg|_{\epsilon = 0} = V \left(\frac{V}{W} \right)^{2n-1} J, \tag{2}$$

where

$$J = \sum_{\{\Gamma\}} \prod_{\{i_\Gamma\}} \alpha_i , \tag{3}$$

$\{\Gamma\}$ is the set of oriented paths from site 1 to site N (see the path in Fig. 1), $\{i_\Gamma\}$ are the sites of this path other than 1 and N, and $\alpha_i = \pm 1$ for $\epsilon_i = \pm W$. We have ignored paths with returns to the left, since they contain additional powers of $V/W \ll 1$. The problem of calculating I models a situation typical of hopping conductivity with a variable hopping length,[3] in which an electron hopping between distant states with energies close to the Fermi level μ is scattered "en route" by a large number of other impurities. According to Ref. 3, the hopping probability and the corresponding inverse resistance of the grid of Miller and Abrahams are proportional to $|I|^2$. The probability x corresponds to the fraction of impurities with $\epsilon_i < \mu$, which is determined by the degree of compensation.

We have calculated J numerically for various values of x and $n \leqslant 100$. Analysis of 2000 realizations of the $\{\alpha_i\}$ data file shows that the shape of the distribution function of the values of J changes with increasing x. While at small values of x the condition $J > 0$ holds in most of the realizations, beginning at $x = x_c = 0.05$ the probabilities for positive (p_+) and negative (p_-) values of J become equal. Figure 2 shows $\Delta p = p_+ - p_-$ versus x for $n = 100$. The function $\Delta p(x)$ does not depend on n at $n > 20$; i.e., it corresponds to the limit $n \to \infty$. The shape of this function is evidence of a second-order phase transition. At $x < x_c$, we can thus predict the sign of the Green's function $G_{1N}(\epsilon = 0) = J(-\epsilon_1)^{-1}(-\epsilon_N)^{-1}$ with some confidence, knowing only the signs of ϵ_1 and ϵ_N. At $x > x_c$, the two signs of the Green's function are equiprobable. The origin of the transition can be seen by comparing the average value of J over the realizations and the fluctuations in J. It is easy to show that we have $\overline{J} = 2^{2n}(1-2x)^{2n}$. This quantity is the average extent to which the number of "positive" paths exceeds the number of "negative" paths in sum (3). At large values of n,

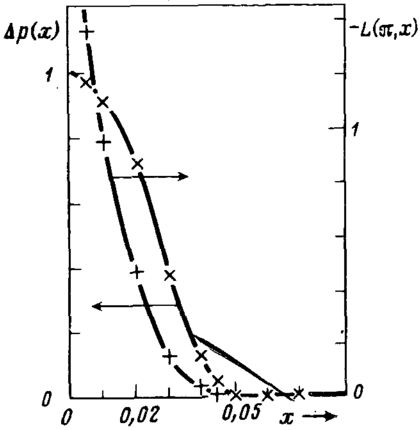

FIG. 2.

this quantity is much smaller than the total number of paths, $\simeq 2^{2n}$. Consequently, an average of 2^{2n} paths cancel out in sum (3). If these paths are assumed to be independent (clearly an incorrect assumption), the fluctuations of J would have to be on the order of $D \simeq 2^n$. We see that in the limit $n \to \infty$ we have $(\overline{J}/D) \to \infty$ if $2(1-2x) > \sqrt{2}$ or $\overline{J}/D \to 0$ if $2(1-2x) < \sqrt{2}$. This estimate thus gives us a first-order phase transition from $\Delta p = 1$ to $\Delta p = 0$ at $x = (\sqrt{2}-1)/2\sqrt{2} = 0.14$, whereas it is in fact, a second-order transition.

How would this transition be manifested in the Aaronov-Bohm oscillations? For the numerical simulation of this effect, we assumed $\alpha_i = 0$ for all i on a 7×7 square; i.e., we set up an inpenetrable aperture at the solenoid (Fig. 1). The magnetic flux Φ through the aperture is taken into account by multiplying the values of α_i by $e^{i\varphi}$, where $\varphi = 2\pi\Phi/\Phi_0$, on all the sites along the cut starting at the lower corner of the aperture (the dashed line in Fig. 1). According to the spirit of the percolation method,[3] in calculating $\ln[\sigma(\varphi)/\sigma(0)]$ (σ is the conductivity) we should calculate the quantity $L(\varphi,x) = \overline{\ln|J(\varphi)/J(0)|^2}$, where the superior bar means an average over the realizations, and then we should take an average of $L(\varphi,x)$ over the hopping lengths. Figure 3 shows $L(\varphi,x)$ versus φ for various values of x. At $x < x_c$, the magnetoresistance has a period of 2π. With increasing x, $|L(\pi,x)|$ decreases, vanishing at $x = x_c$ (Fig. 2). At $x > x_c$, the magnetoresistance is negative for all φ and has a period of π. The transition to the phase that is disordered in terms of the sign of J thus leads to a replacement of the period Φ_0 by $\Phi_0/2$. In order to explain the relationship between these phenomena, we write J in the form $J = J_1 + J_2$, where J_1 and J_2 are sums over paths passing above and below the solenoid. We then have

$$L(\varphi, x) = \int\int_{-\infty}^{\infty} dJ_1 dJ_2 F(J_1, J_2) \ln \left| \frac{J_1 + J_2 e^{i\varphi}}{J_1 + J_2} \right|^2, \tag{4}$$

where $F(J_1, J_2)$ is the distribution function of J_1 and J_2. For $x = 0$ we have $F(J_1, J_2) = \delta(J_1 - J/2)\delta(J_2 - J/2)$ and $L(\varphi,0) = \ln[(1+\cos\varphi)/2]$, in accordance with Fig. 3. With increasing x, $F(J_1, J_2)$ gradually becomes symmetric in terms of the signs of each variable. Beginning at the point $x = x_c$, the function $F(J_1, J_2)$ becomes $F(J_1^2, J_2^2)$;, we find from (4)

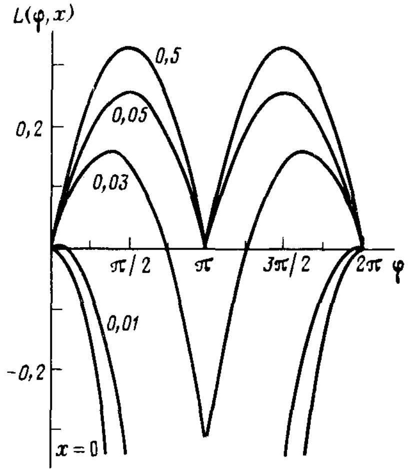

FIG. 3.

$$L(\varphi, x) = 2\int_0^\infty \int_0^\infty dJ_1\, dJ_2 F(J_1^2, J_2^2) \ln\left[1 + \frac{2J_1^2 J_2^2}{(J_1^2 - J_2^2)^2}\left(1 - \cos 2\varphi\right)\right] > 0. \qquad (5)$$

According to (5), at $x > x_c$ the oscillation period is π, and the magnetoresistance is negative for all φ. It can be seen from Fig. 3 that at small values of φ the negative magnetoresistance exists even at $x \ll x_c$. The reason is the large positive contribution to (4) from cases with $|J_1 + J_2| \lesssim |\varphi|(|J_1| + |J_2|)$. These contributions lead to $L(\varphi, x) \simeq |\varphi|$. Clearly, the negative magnetoresistance which we are observing is not a consequence of a suppression of localized corrections to the conductivity due to the circumvention of closed loops in two directions, since our model ignores loops altogether.

We believe that incorporating paths with returns, as we must as we approach the metal-insulator transition, should not liquidate the sign transition but simply shift it. Since the hopping lengths can be on the order of microns near the metal-insulator transition, the possibility of observing the effects predicted here does not seem unrealistic. Experiments might be carried out on a thin-walled cylinder[2] or on films with periodic arrays of identical apertures.[4] In the former case, the hopping length should be greater than the diameter of the cylinder, and in the latter it should be on the order of the diameter of an aperture.

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